The Ahuraic Framework as a Proposed Tripartite Architecture for Organization, Change, and Stability: Integrating CIS, D-CIS, and the Capacity–Attractor Principle

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Abstract

Complex systems are often described through domain-specific measures such as entropy, free energy, correlation length, network metrics, and information-theoretic quantities. This paper presents the Ahuraic Framework, a tripartite architecture for organization, change, and stability based on three constructs: Informational Structural Cost (CIS), its dynamic extension D-CIS, and the Capacity–Attractor Principle (CAP). CIS assigns a scalar structural cost to an admissible configuration via a predecessor-independent generative expansion ratio $r(X) > 0$, and a conditional representation theorem yields

$$\mathrm{CIS}(X) = \ln r(X) + \kappa\, \rho(X).$$

Here $\rho(X)$ is Organizational Rank (hierarchical depth) and κ is a representation-dependent coefficient. D-CIS describes changes in CIS along trajectories, distinguishing the instantaneous rate from the finite change ΔCIS. CAP defines a dimensionless capacity ratio $C = J/J_{\mathrm{crit}}(\rho(X))$ and a statistical persistence hypothesis

$$P(\text{persistence} \mid C > 1) > P(\text{persistence} \mid C < 1),$$

not a deterministic threshold. The paper positions these constructs within the roles of Structural Order, Generativity, and Coherence, with Structure constitutively prior to Organization. A qualitative comparative analysis of 120 heterogeneous cases serves as exploratory motivation. Testable implications, falsification conditions, and two reproducible network case illustrations are provided. The framework is proposed as a substrate-agnostic vocabulary, not a universal physical theory.

Keywords: Ahuraic Framework; Informational Structural Cost; D-CIS; Capacity–Attractor Principle; structural organization; complex systems.

Note on Terminology

The term *Ahuraic* is retained solely as the proper name of the theoretical architecture presented in this article. It carries no cultural, philosophical, metaphysical, theological, or religious premise. The operative scientific vocabulary of the formal core is:

- Structural Order,

- Structural Generativity,
- Structural Coherence.

For continuity with earlier documents, a terminological mapping is provided in Appendix C; those earlier terms do not appear in the formal core. CIS, D-CIS, and CAP refer exclusively to the mathematical constructs defined in this article. NSQ stands for Natural Structural Quantities, an exploratory qualitative comparative protocol (Section 9; Jalali et al. 2026a); it is not to be confused with other uses of the abbreviation in the literature, such as *Nursing Science Quarterly* or the Neuroticism Scale Questionnaire. The following notational conventions are used:

- $(\mathcal{C}, \preceq)$ denotes the structural configuration space with its refinement order;
- G denotes the structural generativity relation;
- $\rho(X)$ denotes the Organizational Rank, equivalently the hierarchical depth of a configuration;
- κ denotes the representation-dependent coefficient multiplying the organizational-rank term in the canonical CIS form;
- C denotes the dimensionless capacity ratio in CAP;
- the word *Organization* denotes a process or family of processes of regulation, stabilization, and coordination acting upon structures. It is not used as a synonym for Organizational Rank or hierarchical depth.

Throughout this article, *Structure is constitutively prior to Organization*. This is a methodological priority: every organizational process presupposes an admissible structural substrate, whereas the existence of a structure does not by itself imply the existence of organization. This is the only place where the priority relation is defined; other sections refer back to it.

1 Introduction

The quantitative description of organization, change, and stability in complex systems remains fragmented across disciplines. Thermodynamics provides energy, entropy, and free energy; information theory provides uncertainty and communication measures; dynamical-systems theory provides attractors, bifurcations, and stability; network science provides connectivity, modularity, and centrality; and quantum many-body physics provides correlation length and entanglement entropy. Although each framework is powerful within its own domain, its central observables are defined on different mathematical objects and cannot be transferred from one domain to another without explicit operational mapping (Shannon, 1948; Jaynes, 1957; Nicolis and Prigogine, 1977; Tononi, 2004; Mitchell, 2009). This fragmentation is not merely technical; it reflects a deeper conceptual gap: most existing measures are introduced as domain-specific proxies rather than derived from a common structural foundation.

Several established lines of work have already addressed related problems from different directions. Shannon formalized information and uncertainty over probability distributions, while Landauer connected logical information processing to physical dissipation (Shannon, 1948; Landauer, 1961). Simon analysed hierarchy and near-decomposability in complex systems, and Anderson emphasized the emergence of higher-level descriptions that are not simply reducible to lower-level laws (Simon, 1962; Anderson, 1972). Nicolis and Prigogine, and later Haken, developed self-organization and collective-order descriptions in nonequilibrium and synergetic systems (Nicolis and Prigogine, 1977; Haken, 1983). In biological and organizational theory, Maturana and Varela introduced autopoiesis and used the term structural coupling to describe organism–environment interaction, while more recent constraint-closure accounts formalize bio-

logical organization as mutual dependence among constraints (Maturana and Varela, 1980; Montévil and Mossio, 2015; Lehman and Kauffman, 2021; Wilson and Prescott, 2022). In algorithmic and statistical complexity, Bennett proposed logical depth, and Crutchfield and Young introduced statistical complexity for predictive structure (Bennett, 1986; Crutchfield and Young, 1989). In dynamical-systems theory, attractor capacities and threshold-like transitions have long been studied (Young, 1981; Bak et al., 1987). In cognitive neuroscience, Tononi proposed integrated information, and Friston introduced the free-energy principle for adaptive systems (Tononi, 2004; Friston, 2010). Recent applied work on supply-chain innovation, organizational cost asymmetries, hierarchical coordination ceilings, and dynamic attractors similarly confronts constraints, generative change, and coherence, although usually in domain-specific language (Malacina and Lintukangas, 2025; O'Connell and Onderwaater, 2026; Brandes, 2026).

The present article does not claim that organization, self-organization, structural coupling, constraint closure, complexity, attractors, or information-theoretic cost are new concepts. Each has an established literature. The proposed contribution is architectural: it places admissible structural configurations, generative accessibility, and coherence in a common configuration-space description, and then defines a conditional structural cost (CIS), its trajectory-level extension (D-CIS), and a capacity ratio (CAP) within that space. The novelty claim therefore concerns the integration and formalization, rather than the discovery of any of the underlying phenomena. This convergence suggests that a proposed tripartite architecture—separating structural order, structural generativity, and structural coherence—may provide a useful, formally substrate-agnostic vocabulary, whose operational realizations remain domain-dependent, for organization, change, and stability. The present article develops such an architecture within the Ahuraic Framework. It introduces three formal constructs:

- **CIS** – Informational Structural Cost, which assigns a scalar structural cost to an admissible configuration through a predecessor-independent generative expansion ratio $r(X) > 0$. Under explicitly stated architectural assumptions on a well-founded graded structural configuration space $(\mathcal{C}, \preceq)$, a conditional representation theorem yields the canonical form

$$\mathsf{CIS}(X) = \ln r(X) + \kappa\, \rho(X), \tag{1.1}$$

  where $\rho(X)$ is the Organizational Rank (hierarchical depth) and κ is a representation-dependent coefficient within the selected model class.

- **D-CIS** – Dynamic Informational Structural Cost, which describes changes in structural cost along a trajectory. To avoid ambiguity, the instantaneous D-CIS rate is distinguished from the finite change ΔCIS.

- **CAP** – Capacity–Attractor Principle, which links persistence to a dimensionless capacity ratio $C = J/J_{\mathsf{crit}}(\rho(X))$ together with a statistical persistence hypothesis

$$P(\text{persistence} \mid C > 1) > P(\text{persistence} \mid C < 1),$$

  rather than a deterministic threshold claim.

These constructs are analytical components of a descriptive architecture whose empirical content must be supplied by domain-specific operationalization. The present formulation refines earlier architectural descriptions of the Ahuraic Framework (Jalali and Jalali, 2025a,b) while restricting the formal core to the proposed tripartite architecture and separating the broader interpretive layer (Appendix C) from the conditional mathematical results. The terms *Structure* and *Organization* are used as defined in the Note on Terminology; in particular, Organization is a process, whereas hierarchical depth is represented separately by the Organizational Rank $\rho(X)$.

The approach is motivated by an exploratory qualitative maximum-variation comparative analysis of 120 heterogeneous cases conducted within the Natural Structural Quantities (NSQ) procedure. In the NSQ procedure, the term "case" refers to the unit of analysis and may denote

a system, process, structure, pattern, procedure, diffusion/reproduction process, or a composite thereof. That analysis suggested that descriptions of persistence, transformation, and dissolution repeatedly involve flow-like support, constrained configurations, dynamic compensation, and threshold-like transitions. These recurring descriptors motivate the distinction between structural state, structural change, and structural stability (Section 9).

The central claim of this article is deliberately limited. The Ahuraic Framework is proposed as a common, formally substrate-agnostic descriptive vocabulary for separating constraint, generative change, and coherence across heterogeneous systems, not as a universal physical theory or a replacement for domain-specific models. The framework is falsifiable through operationalization: if the proposed constructs cannot be mapped reproducibly to measurable variables, or if they do not provide predictive discrimination beyond simpler baseline descriptions, the framework should be revised or rejected. The limitations of the formal results and of the case illustrations are collected in Section 12. An optional interpretive vocabulary (Asha, Spenta, Hamakshāni, Āshid, and the Sephere layers) is given in Appendix C; it does not enter the axioms, assumptions, or proofs.

The remainder of the article is organized as follows. Section 2 presents the research method and the declaration of generative AI and AI-assisted technologies. Section 3 defines the Structural Configuration Space. Section 4 introduces three architectural roles: Structural Order, Structural Generativity, and Structural Coherence. Sections 5–7 define CIS, D-CIS, and CAP, respectively. Section 8 presents the integrative architecture connecting the three constructs. Section 9 summarizes the qualitative NSQ motivation. Section 10 states testable implications and falsification conditions. Section 11 summarizes the reproducible case illustrations; full computational details are provided in a linked Zenodo repository. Section 12 discusses limitations and boundaries. Section 13 concludes.

2 Research Method

This study was conducted using a theoretical–analytical approach within a human–AI collaborative workflow. AI-based tools (ChatGPT, DeepSeek, Claude, and Grok) were used only as auxiliary instruments. The human authors were solely responsible for conceptualization, problem formulation, theoretical framework, mathematical derivations, interpretation, and all final scientific decisions. Detailed disclosure of AI use is provided in the Declaration of Generative AI and AI-Assisted Technologies below. In addition to the formal theoretical development, this study includes an exploratory qualitative analysis using the Natural Structural Quantities (NSQ) procedure (Section 9) and two reproducible network case illustrations (Section 11). For the NSQ component, the authors defined the coding framework, conducted the analysis, reviewed and revised all codes, resolved inconsistencies, and determined the final themes and their interpretation. No computational scripts were required or used for this analysis. For the network case studies (Zachary's karate club and Bitcoin OTC), the authors designed the analytical protocols, reviewed the documented computational outputs, and interpreted the findings. This release carries forward computational materials from version 2.1.1 and does not claim a fresh end-to-end rerun of every analysis from raw inputs. In all cases, the human authors defined the protocols, reviewed the materials used in the manuscript, and made the final decisions regarding the inclusion and presentation of findings.

2.1 Declaration of Generative AI and AI-Assisted Technologies

During the preparation of this work, the authors used ChatGPT, DeepSeek, Claude, and Grok as auxiliary instruments within a human–AI collaborative research workflow. The authors were solely responsible for conceptualization, problem formulation, theoretical framework, and all fi-

nal scientific decisions. *Mathematical development.* AI tools supported symbolic representation, exploration of alternative formulations, and the drafting and revision of candidate proof arguments and counterexamples, as well as checks of internal coherence. AI outputs were treated as suggestions rather than authoritative verification. The authors reviewed and revised the AI-assisted material and take responsibility for the final mathematical statements and any remaining errors. *Exploratory qualitative analysis (NSQ).* AI tools assisted with initial coding suggestions and preliminary thematic pattern identification. These outputs were preliminary aids only. The authors defined the coding framework, conducted the analysis, reviewed and revised all codes, resolved inconsistencies, and determined the final themes and their interpretation. All analytical decisions and thematic conclusions are the authors' own. *Network case illustrations.* AI tools assisted with computational processing, script debugging, and visualization. The authors designed the analytical protocols, reviewed the documented computational outputs, and interpreted the findings. The v2.1.2 package carries forward computational materials from version 2.1.1 and does not claim a fresh end-to-end rerun of every analysis from raw inputs. *Literature organization, language editing, and formatting.* AI tools supported literature organization, language editing, and manuscript formatting. These uses were limited to organizational and editorial assistance; all intellectual content and final wording were determined by the authors. In all cases, the human authors designed the protocols, reviewed the AI-assisted materials, and made the final decisions regarding inclusion and presentation. All AI-assisted content was reviewed, revised, and approved by the authors. The authors accept full responsibility for the article. This approach is consistent with recent methodological discussions on human–AI collaboration in research (Jaeger and Costa, 2026; Ho et al., 2025; Wiles, 2025; Yager, 2024; Vaccaro et al., 2024).

3 Structural Configuration Space

The domain on which the three constructs are defined is the Structural Configuration Space. Let $(\mathcal{C}, \preceq)$ denote a set of admissible structural configurations equipped with a refinement relation. A configuration $X \in \mathcal{C}$ is a representation of selected elements and relations under a specified descriptive scheme. The relation $X \preceq Y$ means that Y is an admissible refinement, extension, or structurally enriched successor of X. We restrict attention to the class of configuration spaces satisfying the following well-foundedness and finite-depth conditions:

1. $\mathcal{C}$ contains at least one minimal configuration, i.e. $\min(\mathcal{C}) \neq \emptyset$.
2. For every configuration $X \in \mathcal{C}$ considered in the framework, there exists a finite refinement chain $X_0 \prec X_1 \prec \cdots \prec X_n = X$ with $X_0 \in \min(\mathcal{C})$.
3. Among all such chains, there is at least one chain of minimal length.

Under these conditions, the Organizational Rank of a configuration X is well-defined as

$$\rho(X) = \min\{n \in \mathbb{N}_0 : \exists X_0 \in \min(\mathcal{C}),\ X_0 \prec X_1 \prec \cdots \prec X_n = X\}. \tag{3.1}$$

For a minimal configuration, $\rho(X) = 0$. Thus $\rho(X)$ measures the minimum number of admissible refinement steps required to construct X from a minimal configuration, not from a distinguished predecessor. This minimum-depth definition does not automatically guarantee monotonicity under refinement. The property $X \leq Y \Rightarrow \rho(X) \leq \rho(Y)$ is treated as an empirical hypothesis (H1) in Section 10 rather than as a direct consequence of the definition. Likewise, the additivity $\rho(X \circ Y) = \rho(X) + \rho(Y)$ used as Assumption 4 of Theorem 5.1 is not a consequence of the minimum-depth definition; it is a separate structural condition on the composition operation $\circ$ (see the note following Assumption 4 in Appendix A). We further assume that, after quotienting by structural equivalence, $\leq$ becomes a partial order. Structurally equivalent configurations are identified before the organizational rank is assigned. Consequently, ρ is a function on equiva-

lence classes:

$$X \sim Y \Rightarrow \rho(X) = \rho(Y). \tag{3.2}$$

That is, the rank does not depend on the chosen representative within a structural equivalence class. Three notions must be carefully distinguished:

- *Structure* denotes the admissible arrangement of components and relations.
- *State* denotes a particular physical or computational realization of a structure.
- *Organizational Rank* $\rho(X)$, equivalently Hierarchical Depth, denotes the minimum refinement depth; it is a structural measure, not to be confused with Organization, which is understood as a process of regulation, stabilization, and coherent transformation acting upon admissible structures.

The same physical system may admit multiple structural descriptions, depending on the selected boundary, variables, scale, temporal resolution, and equivalence relation. All structural quantities defined below are therefore description-dependent. This is not a defect; it is a constitutive feature of cross-domain comparison.

4 Three Architectural Roles

Within the Structural Configuration Space $(\mathcal{C}, \leq)$, the framework distinguishes three complementary architectural roles. They are not introduced as new physical forces, but as analytical components of a structural architecture.

4.1 Structural Order (SO)

Structural Order refers to constraints, selection rules, and admissibility relations that restrict or differentiate possible configurations. Formally, SO may be represented by an admissibility indicator or weighting function on a reference space of possible configurations $\mathcal{C}_0$:

$$A_{\mathsf{SO}} : \mathcal{C}_0 \to [0, 1], \tag{4.1}$$

where $A_{\mathsf{SO}}(X) = 0$ indicates exclusion and positive values indicate degrees of admissibility. The admissible configuration space is then

$$\mathcal{C} = \{X \in \mathcal{C}_0 : A_{\mathsf{SO}}(X) > 0\}. \tag{4.2}$$

We require that $\mathcal{C}$ is nonempty and that the admissibility function is specified in advance, rather than chosen after observing the outcome. SO is not equivalent to low entropy or high physical order. A system may have highly restrictive structural rules while still exhibiting substantial microscopic entropy.

4.2 Structural Generativity (SG)

Structural Generativity refers to the capacity or processes through which configurations, distinctions, or structural variations are produced. It may be represented by a generative relation

$$G \subseteq \mathcal{C} \times \mathcal{C}, \tag{4.3}$$

where $(X, Y) \in G$ means that Y can be generated or reached from X under specified rules. The one-step reachable set is

$$R_1(X) = \{Y \in \mathcal{C} : (X, Y) \in G\}. \tag{4.4}$$

In Section 5, this one-step reachable set is identified with the generative-expansion mapping:

$$G(X) = R_1(X). \tag{4.5}$$

Thus, the generative expansion ratio $r(X)$ introduced in Eq. (5.2) is directly based on this relation. We require that $G(X) \neq \emptyset$ and that the relevant structural measure is positive and finite for every configuration in the CIS domain. SG may involve deterministic evolution, stochastic variation, recombination, learning, mutation, or rule-based construction. At the architectural level, the essential property is the existence of a relation by which configurations become accessible from one another.

4.3 Structural Coherence (SH)

Structural Coherence refers to the compatibility, consistency, and integration among structural components, relations, and generated configurations. It ensures that admissible transformations preserve relational consistency and that components can coexist within a coherent whole. A general representation is

$$H : \mathcal{C} \to \mathcal{C}, \tag{4.6}$$

where H denotes a coherence-preserving transformation or a coherence measure, as specified by the domain model. Coherence is not identical to coupling: coherence concerns the overall compatibility and integration of the structural arrangement, whereas coupling concerns the specific dependency relations between components. The symbol H is reserved for the coherence operator, while the coupling operator remains Γ_{SC}. In the minimal formal core of the present paper, SO and SG enter directly through $\mathcal{C}$ and G, whereas SH is treated as an interpretive or regulative role. SH enters formal expressions only through coherence constraints, compatibility conditions, or interaction corrections when those are explicitly included in a domain-specific model. Thus, the three roles are not claimed to be symmetrically present in every formula of the general CIS representation.

Structural Coupling (SC) as a distinct relation. Structural Coupling refers to the dependency relations through which components, processes, constraints, or structural levels mutually influence one another. It is represented by a coupling operator

$$\Gamma_{\mathsf{SC}} : \mathcal{C} \to \mathcal{M}, \tag{4.7}$$

where $\mathcal{M}$ is a space of relational structures, such as matrices, tensors, or hypergraphs. Coupling may be pairwise or higher-order, local or nonlocal, static or time-dependent. SC is not a third architectural role on the same level as SO, SG, and SH; it is a specific structural relation that may contribute to coherence or to interaction corrections in CIS. The three roles (SO, SG, SH) are analytically distinct but are not assumed to be empirically independent. A regulatory interaction, for example, can constrain a state, couple two components, and enable a transition to another configuration. The distinction is therefore functional rather than necessarily material.

5 Informational Structural Cost (CIS)

CIS assigns a scalar structural cost to an admissible configuration. Its derivation, summarized in Appendix A, follows from a set of architectural assumptions: structural invariance, additivity under independent composition, separation of informational and structural contributions, multiplicativity of the generative ratio, additivity of the organizational rank, compositional independent attainability of these two coordinates, and regularity of the component functions. Monotonicity under refinement is not an axiom of the representation theorem; it is treated separately as an empirical hypothesis in Section 10.

5.1 Generative Expansion Ratio

Let $\mathcal{D} \subseteq \mathcal{P}(\mathcal{C})$ be a collection of subsets of the Structural Configuration Space $\mathcal{C}$ that contains all singleton sets $\{X\}$ and all one-step reachable sets considered in the CIS domain. Let $\nu : \mathcal{D} \to \mathbb{R}_{>0}$ be a positive finite set functional. Depending on the operational context, ν may represent a weighted count of admissible configurations, a normalized structural measure, an accessible state weight, or another domain-specific positive measure of structural extent. For a singleton configuration, define the associated structural measure by

$$\mu(X) := \nu(\{X\}). \tag{5.1}$$

Thus $\mu : \mathcal{C} \to \mathbb{R}_{>0}$. Let $G(X) = \{Y \in \mathcal{C} : (X, Y) \in G\}$ denote the one-step admissible generative expansion of X, where $G \subseteq \mathcal{C} \times \mathcal{C}$ is the Structural Generativity relation introduced in Section 4. We require that $G(X) \in \mathcal{D}$, that $G(X) \neq \emptyset$, and that $\nu(G(X))$ be finite and positive for every configuration X included in the CIS domain. The generative expansion ratio is then defined as

$$r(X) = \frac{\nu(G(X))}{\mu(X)} = \frac{\nu(G(X))}{\nu(\{X\})}, \tag{5.2}$$

with $r(X) > 0$. Consequently, $\ln r(X)$ is defined for every configuration included in the CIS domain. This definition is deliberately predecessor-independent. It does not require selecting a distinguished predecessor $\mathrm{pre}(X)$, nor does it depend on a particular refinement path. Instead, it characterizes the generative extent locally associated with configuration X under a specified structural description and generative relation. Different choices of the set functional ν, the admissible configuration space $\mathcal{C}$, or the generative relation G define different operational realizations of CIS. Such choices are not arbitrary after the fact: they must be specified before empirical evaluation and must remain fixed throughout the analysis.

For independently composable configurations X and Y, the theorem requires the multiplicative conditions

$$\nu(G(X \circ Y)) = \nu(G(X))\, \nu(G(Y)), \tag{5.3}$$

$$\mu(X \circ Y) = \mu(X)\, \mu(Y). \tag{5.4}$$

Equivalently, since $\mu(X) = \nu(\{X\})$,

$$\nu(\{X \circ Y\}) = \nu(\{X\})\, \nu(\{Y\}). \tag{5.5}$$

Under these conditions, the generative expansion ratio satisfies

$$r(X \circ Y) = \frac{\nu(G(X \circ Y))}{\mu(X \circ Y)} = \frac{\nu(G(X))\nu(G(Y))}{\mu(X)\mu(Y)} = r(X)\, r(Y). \tag{5.6}$$

Equation (5.6) is an architectural condition used in the conditional representation theorem; in operational applications, it must be tested or justified for the chosen representation of independent composition. Importantly, independent composition is not defined merely by the validity of Eqs. (5.3)–(5.6). In applications, independence must be supported by external structural criteria, such as disjointness of components, absence of cross-component generative dependencies, or separability of admissibility constraints. Otherwise, the additivity and multiplicativity assumptions become circular. The informational contribution to CIS is logarithmic in $r(X)$, whereas the structural contribution is linear in the Organizational Rank $\rho(X)$. The former captures the relative generative extent associated with the configuration, while the latter captures its minimum admissible refinement depth.

5.2 Conditional Representation of CIS

Theorem 5.1 (Conditional representation of CIS). *Let $I : \mathcal{C} \to \mathbb{R}$ be a structural cost functional belonging to the separable class*

$$I(X) = F(r(X)) + H(\rho(X)) + I_0, \tag{5.7}$$

where $I_0 \in \mathbb{R}$ is a constant and $r : \mathcal{C} \to \mathbb{R}_{>0}$ is the generative expansion ratio defined by Eq. (5.2)*. Assume:*

1. Structural invariance. *If $X \sim Y$, then $r(X) = r(Y)$, $\rho(X) = \rho(Y)$, and $I(X) = I(Y)$.*
2. Independent composition. *For all $X, Y \in \mathcal{C}$ for which $X \circ Y$ is defined, $I(X \circ Y) = I(X) + I(Y) - I_0$.*
3. Multiplicative generative coordinate. $r(X \circ Y) = r(X)\, r(Y)$.
4. Additive organizational coordinate. $\rho(X \circ Y) = \rho(X) + \rho(Y)$.
5. Compositional independent attainability. *For every $x, y \in \mathbb{R}_{>0}$ and every $m, n \in \mathbb{N}_0$, there exist independently composable configurations $X, Y \in \mathcal{C}$ such that $(r(X), \rho(X)) = (x, m)$ and $(r(Y), \rho(Y)) = (y, n)$.*
6. Regularity. *F is continuous on $\mathbb{R}_{>0}$. No regularity condition is required for H, whose domain is the discrete set $\mathbb{N}_0$.*

Then, within this separable additive class, the structural cost has the form

$$I(X) = \lambda \ln r(X) + \kappa\, \rho(X) + I_0, \tag{5.8}$$

where $\lambda, \kappa \in \mathbb{R}$. If $\lambda > 0$, then rescaling the cost unit by $1/\lambda$ and shifting the origin by an additive constant yields the normalized representation

$$\mathrm{CIS}(X) = \ln r(X) + \kappa\, \rho(X). \tag{5.9}$$

If $\lambda = 0$, no logarithmic dependence on $r(X)$ appears. If $\lambda < 0$, the interpretation of $r(X)$ as an increasing cost driver would be reversed. Thus, the canonical form in Eq. (5.9) *is conditioned on $\lambda > 0$.*

The proof is given in Appendix A. This is a conditional representation theorem: it characterizes the form of I within the specified separable additive class, under the stated architectural assumptions; alternative functionals outside this class may exist (see Appendix A.4).

Because $r(X) > 0$ and $\rho(X) \in \mathbb{N}_0$, CIS is dimensionless. No assumption is made that CIS must be positive; if $r(X) < 1$, then $\ln r(X) < 0$. CIS should therefore be interpreted as a structural index or structural cost functional, not as a necessarily positive physical cost. The coefficient κ is representation-dependent. It is not a universal constant of nature unless invariance across operational definitions, scales, domains, and measurement procedures has been independently demonstrated. In practical implementations, ρ may be estimated through domain-specific proxies, such as correlation length or entanglement entropy in quantum spin systems. Such proxies do not define ρ; they only provide operational realizations.

5.3 Relation to Existing Measures

CIS is introduced as a structural cost functional defined on the Structural Configuration Space. The adjective *informational* refers to the logarithmic representation of generative multiplicity and does not imply equivalence with Shannon information, thermodynamic entropy, or algorithmic information. This subsection clarifies the relation between CIS and several established quantities that are commonly used as proxies for organization, complexity, or stability.

Information-theoretic quantities.

$$H(p) = -\sum_{x} p(x) \log p(x). \tag{5.10}$$

Shannon entropy is defined on distributions over messages, symbols, or states. CIS is defined on structural configurations and is not a functional of a probability distribution. The logarithmic term in CIS, $\ln r(X)$, measures multiplicative generative expansion, not message uncertainty. Therefore, CIS and Shannon entropy are not equivalent. CIS may, however, admit entropy-like proxies in specific domains after an explicit mapping is supplied. Rényi entropy generalizes Shannon entropy and belongs to the same axiomatic tradition of information measures (Aczél and Daróczy, 1975; Rényi, 1961); CIS remains distinct because it is defined on structural configurations rather than on probability distributions. Landauer's principle connects logical information erasure to physical dissipation. It establishes that logically irreversible operations, such as bit erasure, have a minimum thermodynamic cost. CIS also associates structural cost with configuration-space restriction, but it does not assert that each structural unit has a fixed physical energy cost. The relation between CIS and physical dissipation is an open modeling issue, not an identity. Algorithmic complexity (Kolmogorov complexity) measures the length of the shortest program that generates an object. CIS is conceptually related because both address description or generation, but CIS is not defined on programs or universal Turing machines. Its rank term $\rho(X)$ and expansion ratio $r(X)$ are architectural, not algorithmic.

Complexity and predictive structure. Statistical complexity (Crutchfield and Young, 1989) measures the information stored in the causal states of a process. It is defined within computational mechanics and describes predictive structure. CIS is defined on structural configurations and includes a hierarchical rank term that is not part of statistical complexity. The two quantities are complementary: statistical complexity concerns predictive memory in time series; CIS concerns structural depth and generative multiplicity in configuration space. Logical depth (Bennett, 1986) measures the computational time required to generate an object from a minimal description. Like CIS, it is a generative notion. But logical depth is defined over algorithmic histories; CIS is defined over admissible structural refinement and generative expansion. CIS is not a measure of computational depth. Integrated information Φ_{IIT} (Tononi, 2004) quantifies the irreducible informational integration of a system under a causal partition. It is designed to capture the unity of conscious or complex systems. CIS, by contrast, is a cost assigned to a structural configuration and does not itself define causal integration. Structural Coupling (SC) in the Ahuraic Framework is a distinct dependency relation that may contribute to integration, but CIS is not equivalent to Φ_{IIT}.

Thermodynamic and statistical quantities. Thermodynamic entropy measures the multiplicity of microstates compatible with macroscopic constraints. CIS is not thermodynamic entropy. In some physical realizations, $\rho(X)$ or $\ln r(X)$ may correlate with thermodynamic entropy-like quantities, but such correlations must be derived, not assumed. Free energy is the thermodynamic potential governing equilibrium ensembles. CIS is a structural cost, not an energetic potential. A correspondence between CIS and free energy can exist only if an explicit affine relation between CIS and physical energy is established within a specified operational model. Without such a relation, CIS and free energy remain distinct.

Network and spatial observables. Network modularity and related graph measures describe topological organization. They are domain-specific and graph-dependent. CIS is defined in a substrate-agnostic manner, although operational realizations remain domain-dependent. Graph measures may serve as operational proxies for ρ or for coupling structure, but they do not define

CIS. Correlation length and entanglement entropy are physical observables in quantum many-body systems. In earlier operational work, they were used as possible proxies for Organizational Rank ρ. They are not identical to ρ, and their validity is regime-dependent.

Self-organization and organizational closure. Self-organization theory (Nicolis and Prigogine, 1977; Haken, 1983) explains how macroscopic order emerges from microscopic interactions under nonequilibrium conditions. SO/SG/SH provide an architectural decomposition at a different descriptive level. CIS is a cost over structural configurations, not a thermodynamic production or order-parameter equation. Autopoiesis (Maturana and Varela, 1980) uses structural coupling to describe organism–environment interaction. The Ahuraic Framework generalizes coupling as a distinct structural relation, SC, applicable beyond biological systems. It does not claim to have introduced the term or concept; it reuses it in a substrate-agnostic description. Coherence, by contrast, is represented by the architectural role SH and should not be identified with coupling. Constraint closure (Montévil and Mossio, 2015; Wilson and Prescott, 2022) formalizes biological organization as mutual dependence among constraints. The present framework's SO/SG/SH can be seen as a broader architectural decomposition that includes constraint closure as a specific class of coupled constraint structures. CAP, in particular, may be used to ask when such constraint structures can persist under a given enabling flux.

Free-energy and attractor principles. Friston's free-energy principle (Friston, 2010) describes adaptive systems as minimizing variational free energy. CAP is different: it is a capacity-to-persistence condition, not a variational inference principle. CAP does not require a generative model or Bayesian agent. The term *attractor* in CAP refers to a basin of admissible structural configurations, not to predictive coding or active inference. Attractor capacity in dynamical systems (Young, 1981) refers to a specific mathematical notion of capacity associated with attractors. The present use of the term capacity is operational and resource-based; it is not identical to the topological or metric capacity studied in that literature and should not be confused with it.

The explanatory value of CIS depends on its ability to organize such proxies within a common architectural description, not on reducing them to a single physical observable.

6 Dynamic Informational Structural Cost (D-CIS)

CIS is a state functional. It assigns a scalar value to a single admissible configuration, but it does not by itself determine how that value changes over time. The Dynamic Informational Structural Cost extends CIS to trajectories through the Structural Configuration Space. D-CIS is a derived quantity rather than an independent construct on the same footing as CIS and CAP. It is introduced only to describe changes in CIS along an already specified trajectory. To avoid terminological ambiguity, we distinguish two related but distinct notions:

- D-CIS rate – the instantaneous rate of change of CIS along a trajectory;
- ΔCIS – the finite change of CIS over a time interval or across a discontinuous transition.

6.1 Instantaneous D-CIS Rate

The Organizational Rank $\rho : \mathcal{C} \to \mathbb{N}_0$ is intrinsically discrete in the present formulation. Consequently, the finite-change formulation introduced in Section 6.2 is the primary form of D-CIS for structural transitions, refinements, and observational settings in which configurations are recorded at discrete times. An instantaneous rate is nevertheless useful when the structural trajectory admits a continuous representation, or when a continuous, domain-specific surrogate

for Organizational Rank has been defined. Let $X(t) \in \mathcal{C}$, $t \in [t_0, t_1]$, denote a structural trajectory. Suppose that $r(X(t))$ is differentiable and that a continuous surrogate $\tilde{\rho}(X(t)) \in \mathbb{R}_{\geq 0}$ has been explicitly defined for the discrete rank $\rho(X(t))$. Such a surrogate may arise through coarse-graining, interpolation between observed structural states, a continuous hierarchical depth estimate, or another domain-specific approximation. It must not be treated as identical to ρ unless that equivalence has been independently established.

Definition 6.1 (D-CIS rate)**.** *Under these conditions, the instantaneous D-CIS rate is defined as*

$$\textit{D-CIS}_{\mathsf{rate}}(t) = \frac{d}{dt}\big[\ln r(X(t)) + \kappa\, \tilde{\rho}(X(t))\big]. \tag{6.1}$$

Therefore,

$$\text{D-CIS}_{\mathsf{rate}}(t) = \frac{1}{r(X(t))}\frac{dr(X(t))}{dt} + \kappa\, \frac{d\tilde{\rho}(X(t))}{dt}. \tag{6.2}$$

The first term represents the instantaneous change in generative expansion, whereas the second represents change in the continuous surrogate for organizational depth. The rate expression should not be used when rank changes occur as discrete jumps and no justified continuous surrogate is available. In those cases, the appropriate description is the finite change in CIS:

$$\Delta\text{CIS} = \ln\left[\frac{r(X_{t+1})}{r(X_t)}\right] + \kappa\,[\rho(X_{t+1}) - \rho(X_t)]. \tag{6.3}$$

Thus, the finite-difference formulation is not merely a numerical approximation to the continuous rate. For structurally discrete systems, it is the primary representation of D-CIS.

6.2 Finite Change

If the structural configuration changes discontinuously, or if only discrete observations are available, the finite-difference form is used:

$$\Delta\text{CIS}(t) = \text{CIS}(X(t + \Delta t)) - \text{CIS}(X(t)). \tag{6.4}$$

More generally, the endpoint change over the interval $[t_0, t_1]$ is given by

$$\Delta\text{CIS}(t_0, t_1) = \text{CIS}(X(t_1)) - \text{CIS}(X(t_0)). \tag{6.5}$$

When the trajectory is differentiable on $[t_0, t_1]$, this endpoint change is equivalently the integral of the instantaneous rate:

$$\Delta\text{CIS}(t_0, t_1) = \int_{t_0}^{t_1} \text{D-CIS}_{\mathsf{rate}}(t)\, dt. \tag{6.6}$$

Thus: CIS is a structural state functional; D-CIS rate is the instantaneous rate of change of that functional; and ΔCIS is the finite change over an interval.

6.3 Balance Decomposition

When a domain-specific balance representation is available, the instantaneous rate may be decomposed as

$$\text{D-CIS}_{\mathsf{rate}}(t) = P_{\mathsf{in}}(J, I) - P_{\mathsf{out}}(J, I), \tag{6.7}$$

where P_{in} denotes processes that generate or increase structural cost, and P_{out} denotes processes that export, dissipate, or reduce it. This decomposition is a modeling choice, not a theorem of the framework. The specific functional forms of P_{in} and P_{out} must be supplied by the domain-specific model. For the balance decomposition to have empirical content, P_{in} and P_{out}

must be defined independently of the observed D-CIS rate, measured or estimated separately, non-negative where they are intended to represent input and output processes, and dimensionally consistent with CIS per unit time. They should not be obtained solely from residual fitting of the D-CIS trajectory. Without these conditions, Eq. (6.7) is a tautological decomposition of any real-valued rate function and provides no scientific constraint. In stationary or compensated regimes, this balance reduces to

$$P_{\text{in}}(J, I) \simeq P_{\text{out}}(J, I), \tag{6.8}$$

which corresponds to the dynamic-balance motif repeatedly observed in the exploratory NSQ record (Section 9).

6.4 Status of D-CIS

D-CIS is not a law of motion. Knowledge of CIS$(X(t_0))$ does not determine $X(t)$: prediction requires an additional evolution rule, boundary conditions, environmental inputs, and capacity constraints. The framework therefore treats D-CIS as a descriptive variable along an already specified trajectory, not as a generator of that trajectory.

7 Capacity–Attractor Principle (CAP)

The Capacity–Attractor Principle (CAP) connects structural persistence to a dimensionless capacity ratio. In this article, CAP is the name of the proposed architectural principle, while its empirical operationalization is formulated as a statistical hypothesis or criterion. The formal content is therefore a capacity-persistence hypothesis. Let J denote a domain-specific enabling capacity variable, which in many applications may be a flux, throughput, resource supply, or other persistent supporting quantity. Let $J_{\text{crit}}(S(X))$ denote the independently estimated critical value of that variable required to sustain the admissible structural configuration at the structural level described by a prespecified descriptor $S(X)$. In the minimal model used in this paper, $S(X) = \rho(X)$, so that $J_{\text{crit}} = J_{\text{crit}}(\rho(X))$. More generally, $S(X)$ may include additional structural covariates, such as redundancy, coupling, or fragility measures, provided that these are fixed before outcome evaluation. The capacity ratio is defined as

$$C(X, J) = \frac{J}{J_{\text{crit}}(S(X))}. \tag{7.1}$$

We require $J_{\text{crit}}(S(X)) > 0$ for all configurations considered, and we require J and J_{crit} to be expressed in the same units or as dimensionless normalizations thereof, so that C is dimensionless. If J_{crit} were selected after observing the outcome, then the inequality $C \geq 1$ would reduce to the definition $J \geq J_{\text{crit}}$ and would carry no independent empirical content. CAP is therefore scientifically meaningful only when J_{crit} is estimated independently of the persistence outcome it is intended to predict, and when both J and J_{crit} are operationalized before the analysis.

Hypothesis CAP (statistical operationalization). For a specified structural descriptor $S(X)$, let J and $J_{\text{crit}}(S(X))$ be estimated independently of the persistence outcome over a given observational interval. Define C by Eq. (7.1). Then the conditional probability of persistence should satisfy

$$P(\text{persistence} \mid C > 1) > P(\text{persistence} \mid C < 1), \tag{7.2}$$

rather than persistence being a deterministic function of $C \geq 1$. The deterministic threshold $C = 1$ is therefore a limiting idealization. In empirical settings, $C = 1$ should be treated as a decision boundary whose predictive value must be assessed through discrimination measures, calibration, or appropriate baseline comparisons. The criterion $C > 1$ is hypothesized to be associated with a higher probability of persistence, but it is neither universally necessary nor

sufficient, because stability also depends on attractor structure, noise, coupling, and external perturbations. In particular, systems with $C < 1$ may persist temporarily through reserves, inertia, buffering, or hysteresis.

7.1 Attractor Landscape Interpretation

CAP can be embedded in an optional effective attractor landscape. In this representation, the system moves on a landscape whose minima correspond to admissible structural configurations. The term *attractor* refers to the set of configurations toward which a domain-specific dynamical rule tends from a neighborhood. CAP does not itself define the attractor or provide the landscape; those elements must be supplied by a specific dynamical model. CAP only constrains whether an attractor basin can retain structural integrity under a given enabling capacity. The depth and stability of a minimum depend jointly on the effective attractor structure and on the capacity ratio C. A configuration with $C > 1$ may still be unstable if the landscape is flat or if fluctuations are strong. In such a landscape, bifurcations correspond to the creation or loss of stable minima; however, CAP itself is not committed to a specific gradient or bifurcation equation. Coupling can act as negative feedback (stabilizing) or positive feedback (destabilizing); the sign and structure of feedback must be supplied by the domain-specific model.

7.2 Relation to Existing Stability and Transition Concepts

CAP is not presented as a new discovery of threshold phenomena. Critical transitions, tipping points, attractor capacity, and resource-limited persistence are established topics in dynamical-systems theory, ecology, and physics (Scheffer et al., 2009; Young, 1981; Bak et al., 1987). The purpose of CAP is to parameterize such conditions in a structural form that can be applied across domains while keeping the structural level explicit.

Attractor capacity. In dynamical-systems theory, the capacity of an attractor may refer to specific mathematical notions of dimension or capacity associated with attractors (Young, 1981). CAP uses the word "capacity" in an operational, resource-based sense and does not replace geometric or dynamical attractor analysis. If an attractor basin is supplied by an external model, CAP asks what enabling capacity is required to keep the system within that basin. The ratio C thus combines attractor structure and resource constraints, but the formal CAP does not itself define that attractor structure.

Self-organized criticality and threshold transitions. In self-organized criticality, systems evolve to a critical state characterized by power-law avalanches and threshold-like responses (Bak et al., 1987). CAP is broader: it does not assume self-organization to a critical point, but only that a critical value $J_{\text{crit}}(S(X))$ can be defined for the structural level $S(X)$. When J approaches J_{crit}, transitions may become more frequent or more probable, but CAP itself does not specify the statistical form of those transitions.

Free-energy principle. Friston's free-energy principle describes adaptive systems as minimizing variational free energy under a generative model (Friston, 2010). CAP is not a variational principle; it does not require Bayesian inference, prediction, or a model of the environment. A system may satisfy CAP while not being an adaptive agent in the free-energy sense, and vice versa.

Carrying capacity and resource limits. In ecological and social systems, carrying capacity and resource constraints are well studied. CAP is a formal restatement of a similar idea at the structural level: the critical value $J_{\text{crit}}(S(X))$ is the structural analogue of a resource threshold, but it is indexed by the structural descriptor $S(X)$ rather than by population size or biomass alone. This indexing is the main architectural addition of CAP.

Upper critical value. The present formulation uses a lower critical value only. In some systems,

an upper critical value or excess-capacity regime may also be relevant, but this possibility is not developed here. The definitions of J and J_{crit} are domain-specific, and the threshold $C = 1$ is a testable phenomenological criterion, not a derived constant. The scientific value of CAP lies in its ability to provide a common dimensionless parameterization for comparing persistence conditions across heterogeneous systems, provided that J and $J_{\text{crit}}(S(X))$ can be operationalized and validated independently.

7.3 Operational Conditions for CAP

For CAP to possess empirical content, the following operational conditions must be satisfied:

1. J must be operationally defined before outcome evaluation.
2. $J_{\text{crit}}(S(X))$ must be estimated independently of the outcome being predicted.
3. The structural descriptor $S(X)$ must be specified a priori, not chosen after observing the outcome.
4. The threshold $C = 1$ must be tested against appropriate null and baseline models, and the optimal cut-point, if used, must be assessed out-of-sample rather than assumed equal to 1.
5. Uncertainty in both J and J_{crit} must be propagated into uncertainty in C.
6. J and J_{crit} must be expressed in consistent units or as compatible dimensionless quantities.

If these conditions are not met, CAP reduces to a definitional restatement and carries no independent empirical content.

8 Integrative Architecture

The three constructs are best regarded as complementary rather than strictly nested. CIS is a state functional on $\mathcal{C}$. D-CIS is its change along a specified trajectory. CAP is a capacity-persistence hypothesis based on a prespecified structural descriptor, which in the minimal model is $\rho(X)$. CAP does not directly use CIS or $r(X)$; its connection to the other constructs is through the shared structural descriptor $\rho(X)$. The three constructs therefore provide a descriptive layer that must be supplemented by a domain-specific evolution rule. They do not form a closed dynamical system, and CAP does not generate or constrain trajectories in the sense of a formal dynamical law. The three architectural roles introduced in Section 4 are connected to these constructs as follows:

- Structural Order (SO) determines the admissible configuration space and the constraints that restrict possible states. It therefore fixes the domain over which CIS is defined:

$$\text{SO} : \mathcal{C}_0 \to \mathcal{C}. \tag{8.1}$$

- Structural Generativity (SG) determines the generative relation G and, through the one-step reachable set $G(X)$, the generative expansion ratio $r(X)$. It thus contributes to the informational term of CIS and to the accessibility of configurations along a D-CIS trajectory:

$$\text{SG} : X \mapsto r(X), \qquad G \subseteq \mathcal{C} \times \mathcal{C}. \tag{8.2}$$

- Structural Coherence (SH) is primarily a regulative or interpretive role. It ensures compatibility and consistency among components and generated configurations. In domain-specific models, it may enter through coherence constraints, compatibility conditions, or interaction corrections:

$$\text{SH} : X \mapsto H(X), \qquad H : \mathcal{C} \to \mathcal{C}. \tag{8.3}$$

In the general CIS representation, SH does not appear as an explicit term unless interaction corrections are included. Its absence from the base formula is a statement that the minimal formal core focuses on r and ρ. In addition, the distinct relation of Structural Coupling (SC) determines whether the cost of a composite configuration factorizes or requires an interaction correction. In the independent case, the additive composition condition of Theorem 5.1 holds. In the strongly coupled case, an interaction term I_{int} may appear, and strong coupling may modify the critical value $J_{\text{crit}}(S(X))$ and therefore the CAP condition:

$$\text{SC} : X \mapsto \Gamma_{\text{SC}}(X), \qquad \Gamma_{\text{SC}}(X) \in \mathcal{M}. \tag{8.4}$$

Thus, SO, SG, and SH are the three architectural roles, but only SO and SG directly participate in the base CIS formula. SC is a specific dependency relation that may affect coherence and interaction corrections. A diagrammatic summary is:

$$\text{SO} \to \text{SG} \to \text{CIS} \to \text{D-CIS};$$
$$\text{SH (interpretive/coherence)}; \quad \text{SC (interaction/coupling)};$$
$$\text{CAP (linked through the structural descriptor } \rho).$$

The arrows are conceptual, not dynamical laws. The framework does not assert that real systems evolve by gradient descent on CIS, nor that CAP is the only stability condition. These constructs are proposed as complementary descriptive tools for separating state, change, and stability in heterogeneous systems.

9 Qualitative Motivation from the NSQ Comparative Record

The Natural Structural Quantities (NSQ) procedure was an exploratory, qualitative comparative protocol applied to 120 heterogeneous cases (Jalali et al., 2026a). The full NSQ comparative record comprised systems, processes, structures, patterns, procedures, and diffusion/reproduction processes. Sampling covered physical and cosmological, chemical and molecular, biological, cross-domain dynamical, social-economic-cultural, and technological or artificial domains. The same five questions were applied to each case: origin, persistence, transformation, dissolution, and scaling or complexification. Across this purposive maximum-variation sample, four broad categories recurred:

1. *Flow or throughput*: the movement of energy, matter, information, resources, or signals.
2. *Constrained structural configuration*: restricted arrangements, relational patterns, and compatibility conditions.
3. *Dynamic compensation or balance*: the interaction of production and degradation, input and output, or integration and disruption.
4. *Threshold-like transformation*: abrupt or nonlinear changes when a relevant support, coupling, or control variable crosses a domain-specific critical range.

These descriptive categories are not identical to CIS, D-CIS, or CAP. They do not supply the functional form of CIS, the value of κ, the definition of $J_{\text{crit}}(\rho)$, or the statistical threshold $C = 1$, and they do not imply the architectural assumptions of Theorem 5.1—additivity, multiplicativity of $r(X)$, or independent attainability of the structural coordinates. Those assumptions must be established separately for any given operational realization. The NSQ analysis is used only as hypothesis-generating motivation: it suggests that a proposed tripartite architecture should include a structural state descriptor, a trajectory descriptor, and a capacity/stability condition. The correspondence between the four qualitative categories and the formal constructs is loose and interpretive. A mapping table is provided in Appendix D; it is a qualitative coding record whose purpose is transparency, not statistical inference.

A focused re-examination of the five core NSQ questions in a subset of the NSQ study revealed an exploratory empirical motif: systems coded as "actively adaptive" also tended to be coded as exhibiting "signal processing." This qualitative co-occurrence was observed most consistently in higher-complexity domains, with some exceptions in biological-organismal, ecological, and informational cases. No contingency table or inter-coder agreement statistics were computed for this exploratory observation. The protocol suite, sample composition, and initial five-question data summary are documented separately (Jalali et al., 2026a).

10 Testable Implications and Falsification Conditions

The architecture generates several classes of testable hypotheses. Two of these, H1 and H2, correspond to conditions used in Theorem 5.1. However, their role here is different: Theorem 5.1 is a conditional representation theorem. It says that if additivity, multiplicativity, and compositional independent attainability hold, then CIS has the logarithmic-plus-linear form. H1 and H2 are therefore not predictions derived from the theorem; they are empirical tests of whether a given operational realization belongs to the theorem's admissible class. If H1 or H2 fails systematically in a domain, the conditional representation does not apply to that domain, but the theorem itself is not refuted. Throughout this section:

- $\mathsf{CIS}(X)$ denotes the canonical form

$$\mathsf{CIS}(X) = \ln r(X) + \kappa\, \rho(X), \tag{10.1}$$

  corresponding to Eq. (5.9);

- $J_{\mathsf{crit}}(S(X))$ denotes the critical enabling capacity indexed by a prespecified structural descriptor $S(X)$; in the minimal model, $S(X) = \rho(X)$;
- C denotes the capacity ratio defined in Eq. (7.1).

H1 — Monotonicity under refinement. If $X_1 \preceq X_2$, then for any operational realization that satisfies the assumptions of Theorem 5.1 and uses a pre-registered rank and expansion ratio,

$$\mathsf{CIS}(X_1) \leq \mathsf{CIS}(X_2) \tag{10.2}$$

should hold. This is an empirical hypothesis, not an axiom of Theorem 5.1. A sufficient condition for H1 is given in Corollary A.2: both $r(X)$ and $\rho(X)$ must be nondecreasing under refinement, and $\kappa \geq 0$. Systematic violations, after checking the operational definitions, would challenge the operationalization of ρ, the generative mapping G, the set functional ν, the singleton measure μ, or the sign and value of κ.

H2 — Empirical additivity under independent composition. For configurations that have been independently established to be compositionally independent,

$$\mathsf{CIS}(X \circ Y) \simeq \mathsf{CIS}(X) + \mathsf{CIS}(Y). \tag{10.3}$$

This hypothesis tests whether a given operational realization satisfies the additive-composition condition assumed in Theorem 5.1. Persistent non-additivity in demonstrably independent compositions would count against the applicability of the separable functional class in that domain.

H3 — D-CIS balance relation. In systems where a decomposition of the form

$$\text{D-CIS}_{\mathsf{rate}}(t) = P_{\mathsf{in}}(J, I) - P_{\mathsf{out}}(J, I) \tag{10.4}$$

is defined—corresponding to Eq. (6.7)—changes in CIS should be attributable to identifiable generative input and output processes. Since the balance relation is a modeling choice, H3 is an adequacy test for the domain-specific model, not a test of the abstract framework.

H4 — CAP threshold discrimination. Independently estimated J and $J_{\text{crit}}(S(X))$ should predict persistence or transition better than appropriate null and baseline models. The empirical form of CAP is

$$P(\text{persistence} \mid C > 1) > P(\text{persistence} \mid C < 1), \tag{10.5}$$

corresponding to Eq. (7.2), with

$$C = \frac{J}{J_{\text{crit}}(S(X))}. \tag{10.6}$$

In the minimal model, one sets $S(X) = \rho(X)$. The threshold $C = 1$ is useful only if it provides predictive discrimination beyond trivial normalization. If Eq. (10.5) fails when J_{crit} is estimated independently and the operational conditions of Section 7.3 are satisfied, then CAP is not supported in that domain.

H5 — Proxy stability. If a domain-specific proxy $\hat{\rho}$ for Organizational Rank is calibrated in one regime, its validity should be stable across related regimes. For example, in regimes where a reference rank ρ_{ref} is available, one may test

$$|\hat{\rho} - \rho_{\text{ref}}| \leq \varepsilon, \tag{10.7}$$

where ε is a domain-appropriate tolerance. Instability restricts the operational scope of the proxy and indicates that it is not a reliable realization of ρ.

Falsification conditions. To clarify the logical status of possible failures, we distinguish four levels of falsification:

1. *Operationalization failure*: a specific choice of ρ, r, J, J_{crit}, or the associated measures fails to reproduce or discriminate. This does not refute the general architecture.
2. *Domain inapplicability*: the architectural assumptions (additivity, multiplicativity, independent attainability) are systematically violated in a given domain, so the conditional representation theorem does not apply there.
3. *Architectural weakening*: the three-role decomposition or the proposed tripartite structure fails to provide any added descriptive or predictive value across multiple independent domains.
4. *Mathematical refutation*: the proof of Theorem 5.1 or Theorem B.7 is shown to be invalid (not merely that its antecedents are not met in an application).

With this distinction, the framework would be weakened or rejected if:

- CIS could not be defined consistently except through arbitrary parameter choices;
- SO, SG, and SH could not be distinguished operationally in any nontrivial application;
- proposed proxies failed to discriminate the constructs from conventional observables;
- CAP did not outperform simple baseline models;
- different investigators could not reproduce structural classifications using the same criteria;
- H1 and H2 failed systematically across domains that had been independently classified as satisfying the architectural assumptions.

These conditions specify the boundaries of scientific usefulness.

11 Reproducible Case Illustrations

Two computational illustrations show how the quantities of Sections 3–6 can be evaluated on documented network data. They have different roles and are not interchangeable measurements

of canonical CIS. Neither provides statistical validation of CIS, D-CIS, or CAP. Scripts, configuration files, parameter tables, derived outputs, sensitivity analyses, baseline/null analyses, and reproducibility instructions are provided in the Zenodo record `https://doi.org/10.5281/zenodo.23218632`. The raw Bitcoin OTC input dataset is not included; the package documents its source and expected SHA-256 checksum, so a full rerun from raw input requires obtaining that dataset separately and verifying the checksum. The computational environment was Python 3.13.5, NetworkX 3.6.1, NumPy 2.3.5, pandas 2.2.3, and scikit-learn 1.8.0 where applicable. Both main trajectories are deterministic, with ties in edge betweenness broken by the lexicographically smallest edge.

11.1 Zachary's karate club: canonical CIS along a Girvan–Newman trajectory

Configuration space. The graph is undirected and unweighted (34 nodes, 78 edges). The trajectory $X_0, X_1, \dots$ is generated by repeatedly deleting the edge of maximal edge betweenness (Girvan–Newman) until the graph first disconnects; this gives 11 recorded configurations ($k = 0, \dots, 10$). This trajectory is not an empirical realization of the refinement order assumed in the abstract configuration-space definition: edge deletion is a graph coarsening operation. It is used only as a documented trajectory on which the CIS expression is evaluated.

Quantities. The rank proxy is $\rho(X_k) = k$, the number of deletions along this particular trajectory. It is a case-specific proxy, not the minimum refinement depth of Section 3, and edge deletion is a coarsening rather than a refinement. Let $Q(\cdot)$ be the modularity of the connected-component partition ($Q = 0$ for a connected graph), $\mu(X) = 1+Q(X)$, and let $G_\eta(X)$ be the set of graphs obtained by deleting one edge whose betweenness is at least η times the maximum, with $\eta = 0.95$. Set $\nu(G_\eta(X)) = \sum_{Y \in G_\eta(X)} [1 + Q(Y)]$ and $r = \nu/\mu$. Then $\mathrm{CIS}(X_k) = \ln r(X_k) + \kappa\rho(X_k)$ with $\kappa = 1$. The values η, κ, and the normalization were fixed before the analysis and were not tuned to the historical labels. The composition conditions (5.3)–(5.5) are not tested here.

Results. The terminal partition has sizes 15 and 19 and agrees with the historical factions for 32 of 34 members (ARI = 0.7717, NMI = 0.7324, FMI = 0.8832).

Baseline. The partition is produced by the Girvan–Newman rule alone: plain Girvan–Newman gives the identical partition, as do lexicographically-first, lexicographically-last, and 100 random tie-breaking rules (all 32/34, ARI = 0.7717). CIS plays no role in selecting edges or partitions, so this agreement carries no information about CIS.

Sensitivity. At $\eta \in \{0.95, 1.00\}$, $\ln r = 0$ in 9 of the 11 configurations; at $\eta = 0.90$ in 8. Until the final step the candidate successors are connected, $Q = 0$, and r equals the number of candidate edges. The monotonicity results are: $\eta = 0.90$ gives no/no/no/yes/yes for $\kappa = 0, 0.25, 0.5, 1, 2$; $\eta = 0.95$ gives no/no/yes/yes/yes; $\eta = 1.00$ gives no/no/yes/yes/yes. The smallest sufficient κ is 0.2967 for $\eta \geq 0.95$ and 0.6931 for $\eta = 0.90$. Thus monotonicity is parameter-dependent and is driven mainly by the rank term: for $\kappa \geq 0.25$ the correlation of $\mathrm{CIS}(X_k)$ with k is at least 0.96, and $|\mathrm{CIS} - k| \leq \ln 2$ everywhere. The illustration therefore establishes operational computability, not construct validity: it shows that r and ρ can be computed on a graph trajectory, not that CIS carries structural information beyond the trajectory index.

11.2 Bitcoin OTC: partition-level descriptors (not canonical CIS)

Data and subgraph. The Bitcoin OTC web-of-trust data contain 35,592 ratings among 5,881 users. Positive ratings were projected to an undirected weighted graph (5,573 nodes, 18,591 edges; largest component 5,551 nodes, 18,576 edges). The analysed subgraph is the largest k-core with 50–200 nodes under a pre-specified rule, $k = 20$: 102 nodes, 1,581 edges. Girvan–Newman uses unweighted edge betweenness.

Quantities. The principal quantities are the retained internal edge-weight fraction and the

component concentration,

$$R_W(P) = \frac{\sum_{C \in P} W(C)}{W(G)}, \qquad R_C(P) = \sum_i \left(\frac{|C_i|}{n}\right)^2.$$

Neither is identified with $\ln r(X) + \kappa\rho(X)$, and R_C is not the generative expansion ratio $r(X)$. No ARI/NMI/FMI or CAP test is computed because no external labels and no independently specified J, J_{crit}, or persistence outcome exist.

Results and interpretation. The first disconnection occurs at step 26 and gives components of sizes 101 and 1. The illustration is retained as a methodological boundary case: it tests whether partition-level descriptors can be reproduced and interpreted without relabeling them as canonical CIS. Both R_W and R_C are non-increasing, but this holds by construction, since deleting edges can only split components. The isolated node has degree 20, the minimum degree of the subgraph (median 26), and all 20 of its edges are among the 26 deleted edges: the first split removes a boundary node and is not community discovery. Across k-cores $k = 16, \ldots, 20$ the first split occurs at steps 42, 37, 22, 27, 26 and always isolates one node of degree 18–20. On 20 degree-preserving rewired graphs the first split is always $[n-1, 1]$ at steps 20–26 (median 22.5), so step 26 is not distinguishable from the null. By contrast, greedy modularity maximisation finds three communities ($Q = 0.200$ unweighted, 0.271 weighted), whereas the first twelve Girvan–Newman levels have $Q \leq 0.0054$.

12 Limitations and Boundaries

This section is the single consolidated statement of the status and limits of the claims; earlier sections refer to it rather than repeating it.

Description dependence. All structural quantities are defined relative to a selected system boundary, variable set, scale, temporal resolution, and equivalence relation. Numerical values obtained under one description cannot automatically be transferred to another.

Conditional status of the CIS representation. The canonical form

$$\mathrm{CIS}(X) = \ln r(X) + \kappa\,\rho(X) \tag{12.1}$$

is obtained only within the separable additive class and under the architectural assumptions stated in Appendix A: a multiplicative generative coordinate, an additive organizational coordinate, compositional independent attainability, and regularity. The independent-attainability assumption is not expected to hold automatically in finite empirical systems; it is a theorem-level condition that must be justified, restricted, or replaced when an operational domain is specified. The theorem therefore does not establish that any real-world system satisfies its hypotheses, and it does not assert uniqueness over all possible structural measures. H1 and H2 are not predictions of the theorem; they are empirical tests of whether a given operational realization belongs to the theorem's admissible class. The coefficient κ is representation-dependent (Section 5).

Proxy and measure dependence. No single empirical observable is claimed to measure ρ, CIS, D-CIS, or CAP universally. Candidate proxies may be valid only within restricted regimes. The numerical value of $r(X)$ depends on the chosen set functional ν, the singleton structural measure μ, and the generative-expansion mapping G. Different operational choices define different but internally consistent versions of CIS.

No universal dynamical law. CIS is a structural state functional. D-CIS describes changes along a given trajectory but does not determine the trajectory itself. CAP is a capacity condition, not a complete stability theory. Its empirical form is the statistical inequality

$$P(\text{persistence} \mid C > 1) > P(\text{persistence} \mid C < 1), \tag{12.2}$$

not a deterministic threshold implication.

Status of the non-vacuous model. The concrete construction in Appendix A shows only that the architectural assumptions are not empty. It is an analytical toy model; it provides no empirical evidence for CIS, D-CIS, or CAP in real systems.

Status of the reproducible case illustrations. The two network case studies of Section 11 are reproducible computational illustrations. The karate illustration demonstrates one operational evaluation of the CIS expression along a coarsening trajectory, whereas the Bitcoin illustration provides complementary partition-level structural descriptors. Neither provides statistical validation of CIS, D-CIS, or CAP, and they do not establish that the selected operational definitions are uniquely correct. In the karate illustration, the agreement with the historical split is determined by the Girvan–Newman ranking trajectory itself; CIS is not used to select the partition. The monotonicity result is sensitive to the operational conventions for η and κ, which were fixed for the illustration rather than inferred from independent outcome data.

Strong-coupling limitation. The additive composition and factorization conditions are assumed only for configurations that are structurally independent under the chosen description. Strongly coupled compositions may require an interaction term

$$I(X * Y) = I(X) + I(Y) + I_{\text{int}}(X, Y), \tag{12.3}$$

which is not determined by the present axioms. The behavior of CAP under strong coupling likewise requires separate domain-specific modeling.

Exploratory status of NSQ. The 120-case analysis is qualitative and hypothesis-generating. It provides no empirical validation for CIS, D-CIS, or CAP.

No claim of replacement or universality. The framework does not replace thermodynamics, information theory, network science, or dynamical-systems theory. It offers a preliminary descriptive level for separating constraint, generative change, and coherence before domain-specific models are introduced. It does not assert that CIS is a universal cross-system invariant, nor that structural coherence implies logical soundness (Appendix B). The interpretive vocabulary of Appendix C does not enter the formal definitions, axioms, or proofs.

Further extensions not treated. Derived structural quantities such as inverse structural temperature, structural resilience, or capacity density; quantum generalizations; additional architectural principles beyond rank, coherence, and coupling; and possible interpretations of NSQ descriptors as independent structural dimensions all lie outside the present proposed architecture. They are noted only as open directions for future research. The exploratory NSQ record suggests the possible existence of a more explicit dynamical core behind the present architecture: the recurrence of flow-limited persistence, threshold-like change, and compensatory balance motivates the conjecture that an effective potential $V(\Psi; J)$, a two-sided flux window

$$J_{\text{min}} < J < J_{\text{max}}, \tag{12.4}$$

and stochastic attractor dynamics may provide a future dynamical extension of CIS, D-CIS, and CAP (Appendix E). The present article deliberately does not incorporate these elements into the formal core, because doing so would amount to proposing a universal law of motion, which would require a separate empirical research program. A possible domain-specific extension for networked flow systems may introduce openness, polarity, and coherence measures; it is likewise not developed here.

13 Conclusion

This article presented the Ahuraic Framework as a proposed tripartite architecture for organization, change, and stability across heterogeneous systems. The three central constructs—CIS, D-CIS, and CAP—provide a formally substrate-agnostic vocabulary for separating structural state, structural trajectory, and structural persistence, while operational realizations remain domain-dependent. CIS is a state functional defined through a predecessor-independent generative expansion ratio $r(X) > 0$. Under a conditional representation theorem, its canonical form is

$$\mathsf{CIS}(X) = \ln r(X) + \kappa\, \rho(X) \tag{13.1}$$

within a specified separable additive class (Appendix A). D-CIS distinguishes the instantaneous rate of change from the finite change ΔCIS. CAP is expressed as a dimensionless capacity ratio

$$C = \frac{J}{J_{\mathsf{crit}}(\rho(X))} \tag{13.2}$$

with an associated statistical persistence hypothesis rather than a deterministic threshold. Section 11 summarizes two reproducible network case illustrations, using Zachary's karate club network and the Bitcoin OTC trust network; they show how the configuration space, Organizational Rank, generative expansion ratio, and finite ΔCIS can be operationalized in documented network data. The qualitative NSQ survey of 120 cases suggests that constraints, generative change, dynamic compensation, and threshold-like transitions are recurrent structural themes across highly different systems. The status and limits of all of these results are stated in Section 12.

The scientific value of the Ahuraic Framework will ultimately depend on whether its architectural distinctions enable reproducible operational models and independent predictions that are not already available from established theories. The present article defines the proposed core of that research program and states its boundary conditions.

Future Research Directions

The present framework remains open to further empirical testing across domains for which publicly available, independently documented datasets exist. Candidate settings include molecular reaction-path datasets such as Transition1x; cosmological halo merger trees from simulations such as IllustrisTNG or Millennium; historical ecological time series with documented regime shifts, including the Hudson Bay lynx–hare record and the Atlantic cod collapse; organizational communication networks with known failure outcomes, such as the Enron email corpus; the complete neuronal connectivity map of *C. elegans*; open high-energy event records from CERN and quantum-computing calibration and error-rate data, where a pre-existing threshold concept offers a natural test of CAP; materials informatics repositories containing phase-transition data; epidemic spread networks with documented control or uncontrolled outcomes; historical linguistic corpora capturing structural language change; gene regulatory networks with mutation and phenotype data; and interbank networks associated with the 2008 financial crisis. In each setting, the operational definitions of $r(X)$, $\rho(X)$, and J_{crit} would need to be specified before analysis.

Such extensions would not require changing the formal architecture; they would test whether the proposed constructs retain descriptive and predictive value beyond the two network illustrations reported in Section 11.

Data Availability Statement

Two supplementary repositories are provided. First, the qualitative protocol suite, sample composition documentation, and initial five-question data summary associated with the NSQ comparative record are publicly archived and citable through Zenodo:

- Jalali, M., Jalali, S., & Jalali, P. (2026). Ahuraic Framework Protocol Suite (13 Protocols), Sample Set A, and Five-Questions Initial Data Summary (Version 1.0) [Data set]. Zenodo. https://doi.org/10.5281/zenodo.22033039

This repository contains the NSQ protocol definitions, sampling documentation, and the qualitative coding summary used in Section 9 and Appendix D. Second, the reproducible network case-study materials for Section 11—including scripts, configuration files, parameter settings, trajectory tables, and derived outputs—are available in a separate Zenodo repository. The raw Bitcoin OTC input dataset is not included and must be obtained separately from the documented source and checked against the expected SHA-256 checksum:

- Jalali, M., Jalali, S., & Jalali, P. (2026). Ahuraic Framework Reproducible Case Studies: Two Network-Based Illustrations (Version 2.1.2) [Software documentation]. Zenodo. https://doi.org/10.5281/zenodo.23218632

No additional numerical simulations or computational validation experiments beyond these documented case illustrations were performed in this theoretical article. All supplementary materials are available under the licenses indicated in the respective Zenodo records.

Author Contributions

Mahdi Jalali (MJ): Conceptualization, methodology, formal analysis, theoretical development, writing–original draft, and visualization. Sediqeh Jalali (SJ): Writing–review and editing, manuscript revision, and language improvement.

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Conflicts of Interest

The authors declare that they have no competing financial or non-financial interests related to this work.

Ethics Statement

The study used theoretical analysis, qualitative comparative review, and publicly available, de-identified secondary network data. It involved no interaction with, or intervention on, human participants or animals, and no individually identifying information was used. Accordingly, institutional ethical approval was not required.

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Appendix A — Conditional Representation of the Informational Structural Cost

This appendix states the formal assumptions under which the canonical form of CIS is derived and gives the proof of Theorem 5.1.

A.1 Structural Configuration Space and Composition

Let $(\mathcal{C}, \leq)$ be the Structural Configuration Space introduced in Section 3. We assume that $(\mathcal{C}, \leq)$ satisfies the well-foundedness and finite-depth conditions stated there. In particular: (1) $\min(\mathcal{C}) \neq \emptyset$; (2) every $X \in \mathcal{C}$ considered in the framework is reachable from some minimal configuration by a finite refinement chain; (3) after quotienting by structural equivalence, $\leq$ is a partial order. Let $\circ : \mathcal{C} \times \mathcal{C} \rightharpoonup \mathcal{C}$ be a partial binary operation representing independent composition. The expression $X \circ Y$ is defined only for configurations that are structurally independent under the chosen description. Strongly interacting compositions are excluded from this operation and, if relevant, require an explicit interaction correction. We assume

$$X_1 \preceq X_2,\ Y_1 \preceq Y_2 \implies X_1 \circ Y_1 \preceq X_2 \circ Y_2. \tag{A.1}$$

This condition ensures that the refinement order and the composition operation do not contradict one another.

A.2 Generative Expansion Measure

The generative expansion ratio requires a measure both for individual configurations and for the one-step reachable set associated with each configuration. Because $G(X)$ is generally a subset of $\mathcal{C}$, rather than a single configuration, a scalar function defined only on $\mathcal{C}$ is insufficient to evaluate the numerator of the ratio. Let $\mathcal{D} \subseteq \mathcal{P}(\mathcal{C})$ be a domain of structurally relevant subsets of $\mathcal{C}$. We require $\mathcal{D}$ to contain $\{X\}$ for all $X \in \mathcal{C}$, and $G(X)$ to belong to $\mathcal{D}$ for all X in the CIS domain. Let ν be a positive finite set functional on $\mathcal{D}$. The interpretation of ν is domain-dependent: it may be a cardinality-like measure, a weighted count, an accessible-volume measure, a normalized probability weight, or another positive measure of structural extent. No specific physical interpretation is imposed by the representation theorem. For individual configurations, define the associated structural measure by

$$\mu(X) := \nu(\{X\}). \tag{A.2}$$

The one-step generative expansion ratio is then

$$r(X) = \frac{\nu(G(X))}{\nu(\{X\})} = \frac{\nu(G(X))}{\mu(X)}. \tag{A.3}$$

We assume that $0 < \nu(\{X\}) < \infty$ and $0 < \nu(G(X)) < \infty$ for every configuration X in the CIS domain. Hence $r(X) \in \mathbb{R}_{>0}$. For independently composable configurations X and Y, the theorem requires the multiplicative conditions

$$\nu(G(X \circ Y)) = \nu(G(X))\,\nu(G(Y)), \tag{A.4}$$

$$\nu(\{X \circ Y\}) = \nu(\{X\})\,\nu(\{Y\}), \tag{A.5}$$

the latter being equivalent to $\mu(X \circ Y) = \mu(X)\mu(Y)$. It follows directly that

$$r(X \circ Y) = \frac{\nu(G(X \circ Y))}{\nu(\{X \circ Y\})} = \frac{\nu(G(X))\nu(G(Y))}{\nu(\{X\})\nu(\{Y\})} = r(X)\, r(Y). \tag{A.6}$$

These assumptions apply only to configurations judged structurally independent under the selected description. If configurations are strongly coupled, the factorization conditions may fail and an interaction correction may be required.

A.3 Assumptions

Let $I : \mathcal{C} \to \mathbb{R}$ be a structural cost functional. We consider functionals of the separable form

$$I(X) = F(r(X)) + H(\rho(X)) + I_0, \tag{A.7}$$

where $I_0 \in \mathbb{R}$ is a constant. The following assumptions are imposed.

Assumption 1 — Structural invariance. If $X \sim Y$, then $r(X) = r(Y)$, $\rho(X) = \rho(Y)$, $I(X) = I(Y)$. Thus, the two structural coordinates and the cost functional are all well-defined on structural equivalence classes.

Assumption 2 — Independent composition. For all $X, Y \in \mathcal{C}$ for which $X \circ Y$ is defined,

$$I(X \circ Y) = I(X) + I(Y) - I_0. \tag{A.8}$$

Assumption 3 — Multiplicative generative coordinate. For independent composition,

$$r(X \circ Y) = r(X)\, r(Y). \tag{A.9}$$

Assumption 4 — Additive organizational coordinate. For independent composition,

$$\rho(X \circ Y) = \rho(X) + \rho(Y). \tag{A.10}$$

Note on Assumption 4. As stated in Section 3, additivity of ρ under independent composition is a separate structural condition on $\circ$ and does not follow from the minimum-depth definition of ρ. Where ρ is identified with minimum refinement depth, additivity must be verified independently; where it is not expected to hold, the theorem applies only with an alternative additive rank coordinate, or with the composition operation restricted accordingly.

Assumption 5 — Compositional independent attainability. For every $x, y \in \mathbb{R}_{>0}$ and every $m, n \in \mathbb{N}_0$, there exist independently composable configurations $X, Y \in \mathcal{C}$ such that $(r(X), \rho(X)) = (x, m)$ and $(r(Y), \rho(Y)) = (y, n)$. This condition guarantees not only that the two coordinates are independently realizable, but also that the realizing configurations can be combined under $\circ$, which is necessary for the separation step in the proof.

Assumption 6 — Regularity. The function $F : \mathbb{R}_{>0} \to \mathbb{R}$ is continuous on $\mathbb{R}_{>0}$. No continuity condition is required for H, whose domain is the discrete set $\mathbb{N}_0$.

Normalization convention. Since F and H are determined only up to additive constants that can be absorbed into I_0, we impose, without loss of generality,

$$F(1) = 0, \qquad H(0) = 0. \tag{A.11}$$

The values $1 \in \mathbb{R}_{>0}$ and $0 \in \mathbb{N}_0$ are both present in the domains, so this convention is well-defined.

A.4 Theorem and Corollary

Theorem A.1 (Conditional representation of CIS)**.** *Under Assumptions 1–6 and the normalization convention above, the structural cost functional belongs to the family*

$$I(X) = \lambda \ln r(X) + \kappa\, \rho(X) + I_0, \tag{A.12}$$

where $\lambda, \kappa \in \mathbb{R}$. If $\lambda > 0$, then rescaling the cost unit by $1/\lambda$ and shifting the origin by an additive constant yields the normalized representation

$$\mathrm{CIS}(X) = \ln r(X) + \kappa\, \rho(X). \tag{A.13}$$

If $\lambda = 0$, no logarithmic dependence on $r(X)$ appears. If $\lambda < 0$, the interpretation of increasing generative expansion as increasing structural cost would be reversed. Thus, the canonical form in Eq. (A.13) *is conditioned on $\lambda > 0$.*

Corollary A.2 (Sufficient monotonicity condition)**.** *Suppose that, in addition to Assumptions 1–6, both coordinates are nondecreasing under refinement: $X \leq Y \Rightarrow r(X) \leq r(Y)$ and $\rho(X) \leq \rho(Y)$. Then the representation in Eq.* (A.12) *is nondecreasing under refinement whenever $\lambda \geq 0$ and $\kappa \geq 0$. In particular, for the canonical normalization $\lambda = 1$, monotonicity holds if $\kappa \geq 0$. This corollary is a sufficient condition only; it does not assert that monotonicity is automatic.*

Scope of the theorem. Theorem A.1 is conditional: it holds within the class of separable, additive functionals satisfying the above architectural assumptions. It does not assert uniqueness over all conceivable structural measures. Functionals with interaction corrections, nonseparable dependencies, or alternative composition rules may lie outside this class and require independent justification.

A.5 Proof

From the separable form (A.7) and the independent-composition condition (A.8),

$$F(r(X \circ Y)) + H(\rho(X \circ Y)) + I_0 = F(r(X)) + F(r(Y)) + H(\rho(X)) + H(\rho(Y)) + I_0.$$

Canceling I_0 and using Assumptions 3–4,

$$F(r(X)r(Y)) + H(\rho(X) + \rho(Y)) = F(r(X)) + F(r(Y)) + H(\rho(X)) + H(\rho(Y)). \tag{A.14}$$

Rearranging,

$$F(r(X)r(Y)) - F(r(X)) - F(r(Y)) = -\big[H(\rho(X) + \rho(Y)) - H(\rho(X)) - H(\rho(Y))\big]. \tag{A.15}$$

By Assumption 5, for arbitrary $x, y \in \mathbb{R}_{>0}$ and $m, n \in \mathbb{N}_0$, there exist independently composable configurations X, Y with $(r(X), \rho(X)) = (x, m)$ and $(r(Y), \rho(Y)) = (y, n)$. Therefore the left-hand side of (A.15) can be varied independently of the right-hand side. Hence each side must be constant. Let this constant be c. Setting $m = n = 0$, the right-hand side becomes

$$H(0 + 0) - H(0) - H(0) = 0 \tag{A.16}$$

because $H(0) = 0$. Thus $c = 0$. Therefore we obtain the two independent functional equations

$$F(xy) = F(x) + F(y), \qquad x, y \in \mathbb{R}_{>0}, \tag{A.17}$$

$$H(m + n) = H(m) + H(n), \qquad m, n \in \mathbb{N}_0. \tag{A.18}$$

Equation (A.17) is the multiplicative Cauchy functional equation. Since F is continuous on $\mathbb{R}_{>0}$ by Assumption 6, its general solution is

$$F(x) = \lambda \ln x. \tag{A.19}$$

Equation (A.18) is the additive Cauchy equation on $\mathbb{N}_0$. Its solution is $H(n) = nH(1)$. Defining $\kappa = H(1)$,

$$H(\rho) = \kappa\rho. \tag{A.20}$$

Substitution into (A.7) gives Eq. (A.12). The normalized form (A.13) follows when $\lambda > 0$, by rescaling the cost unit and shifting the origin. □

A.6 Non-Empty Model Class

The assumptions used in Theorem A.1 are jointly satisfiable. The following elementary construction provides a non-empty model class. Let $\mathcal{C} = \mathbb{R}_{>0} \times \mathbb{N}_0$. A configuration is written as $X = (a, n)$, where $a > 0$ is a positive generative coordinate and $n \in \mathbb{N}_0$ is an organizational-rank coordinate. Define the one-step refinement relation by

$$(a, n) \rightsquigarrow (a, n + 1). \tag{A.21}$$

Let $\leq$ be the reflexive transitive closure of this relation. Equivalently,

$$(a, n) \preceq (b, m) \iff a = b \text{ and } n \leq m. \tag{A.22}$$

This relation is a partial order. The minimal configurations are $\min(\mathcal{C}) = \{(a, 0) : a > 0\}$. The Organizational Rank is defined by

$$\rho(a, n) = n. \tag{A.23}$$

This agrees with the definition of rank as the minimum number of one-step refinements from a minimal configuration. Define independent composition as the binary operation

$$(a_1, n_1) \circ (a_2, n_2) = (a_1 a_2,\ n_1 + n_2). \tag{A.24}$$

Because $a_1 a_2 > 0$ whenever $a_1, a_2 > 0$, the operation is closed on $\mathcal{C}$. Compatibility with the refinement order is immediate: if $(a, n_1) \leq (a, n_2)$ and $(b, m_1) \leq (b, m_2)$, then $(ab, n_1 + m_1) \leq (ab, n_2 + m_2)$. Let $\mathcal{D}$ be the set of all finite non-empty subsets of $\mathcal{C}$. For $S \in \mathcal{D}$, define

$$\nu(S) = \sum_{(a,n)\in S} a. \tag{A.25}$$

This is a positive finite set functional on $\mathcal{D}$. The singleton structural measure is therefore $\mu(a, n) = \nu(\{(a, n)\}) = a$. Define the generative expansion mapping by

$$G(a, n) = \{(a^2, n)\}, \qquad \nu(G(a, n)) = \nu(\{(a^2, n)\}) = a^2. \tag{A.26}$$

(The generative relation G of Section 4 is not required to coincide with the refinement relation; here it is a separate relation on $\mathcal{C}$.) Therefore

$$r(a, n) = \frac{\nu(G(a, n))}{\mu(a, n)} = \frac{a^2}{a} = a, \tag{A.27}$$

and for $X = (a_1, n_1)$ and $Y = (a_2, n_2)$,

$$r(X \circ Y) = r(a_1 a_2, n_1 + n_2) = a_1 a_2 = r(X)\, r(Y), \tag{A.28}$$

$$\rho(X \circ Y) = n_1 + n_2 = \rho(X) + \rho(Y). \tag{A.29}$$

The multiplicative conditions for the set functional are also satisfied:

$$\nu(\{X \circ Y\}) = a_1 a_2 = \nu(\{X\})\,\nu(\{Y\}), \tag{A.30}$$

$$\nu(G(X \circ Y)) = (a_1 a_2)^2 = a_1^2 a_2^2 = \nu(G(X))\,\nu(G(Y)). \tag{A.31}$$

Compositional independent attainability is satisfied because for any $x, y > 0$ and $m, n \in \mathbb{N}_0$, the configurations $X = (x, m)$ and $Y = (y, n)$ are in $\mathcal{C}$, are independently composable, and have the required coordinates. Structural invariance holds trivially because we take structural equivalence to be equality: $X \sim Y \iff X = Y$. Finally, with $I(a, n) = \lambda \ln a + \kappa n + I_0$ we have

$$I(X \circ Y) = \lambda \ln(a_1 a_2) + \kappa(n_1 + n_2) + I_0 = I(X) + I(Y) - I_0. \tag{A.32}$$

All assumptions of Theorem A.1 are therefore satisfied by this model class, so the class of structural systems satisfying Assumptions 1–6 is non-empty. This construction is an analytical toy model, not an empirical model of any physical, biological, social, or technological system; its sole purpose is to demonstrate logical consistency and non-emptiness of the assumptions.

A.7 Scope of the Result

This derivation does not require $r(X)$ or $\rho(X)$ to be directly measurable in all systems. It only fixes the admissible functional form of I under the stated axioms. Operational realizations of ρ and r must be supplied separately for each domain, and their compatibility with Assumptions 3–5 must be checked before applying Eq. (A.12) or Eq. (A.13). Extensions for strongly coupled configurations may require an interaction term $I(X * Y) = I(X) + I(Y) + I_{\text{int}}(X, Y)$, which is not determined by the present axioms.

A.8 Classical Positioning

The proof technique used in Theorem A.1—reduction to Cauchy functional equations under regularity assumptions—is classical in the foundations of information measures and follows the tradition of the characterization of entropy by Shannon, Rényi, and Aczél–Daróczy. A standard reference is Aczél and Daróczy (1975). The present theorem applies the same functional-equation method not to probability distributions, but to structural configurations equipped with a generative expansion ratio and an organizational rank. This positioning clarifies that the formal mechanism is well established, while the domain of application is different.

A.9 Architectural Interpretation (Non-Formal)

In the broader architectural interpretation, Structural Order corresponds to admissibility constraints, Structural Generativity to the generative mapping G, and Structural Coherence to compatibility and integration conditions. Structural Coupling, by contrast, is a distinct dependency relation that may contribute to coherence or to interaction corrections. The quantities r and ρ are formal representations of generative extent and refinement depth, respectively. The framework's interpretive layer can be expressed in neutral academic language as follows. These roles do not appear as separate axioms in Theorem A.1.

Interpretive role	Academic equivalent	Formal counterpart
Admissibility scheme	admissibility predicate	$\mathcal{C}, \leq, A_{\mathsf{SO}}$
Structural constraint	constraint/acceptance role	ρ, refinement relation
Generative expansion	generation/expansion role	G, r
Structural Coherence	compatibility/integration role	H, coherence conditions
Structural Coupling	dependency/interaction role	Γ_{SC}, interaction terms
Minimal distinction	minimal admissible configuration	elements of $\min(\mathcal{C})$
Stable structural unit	stable admissible configuration	configurations with $\rho > 0$ and positive CIS
Relational composite	composite structure	independent composition $\circ$
Higher-order structural composite	higher-order composite	coupled/composite configurations

Appendix B — Admissibility and Coherence Preservation

This appendix formalizes a meta-level admissibility architecture. It is kept separate from the mathematical core of CIS and does not introduce new axioms into Theorem A.1.

B.1 Formal Systems

Let a formal structural system be written as $\mathsf{Sys} = (\mathcal{C}_{\mathsf{Sys}}, R_{\mathsf{Sys}}, G_{\mathsf{Sys}}, I_{\mathsf{Sys}})$, where $\mathcal{C}_{\mathsf{Sys}}$ is the configuration space of Sys; R_{Sys} is the refinement relation; G_{Sys} is the generative relation; and I_{Sys} is the structural interpretation, if any. A system is considered only if its configuration space satisfies the conditions of Appendix A.

B.2 Admissibility Predicate

Admissibility is defined as a meta-level predicate, not as an object inside a particular formal system: $\mathsf{Adm} : \mathsf{Systems} \to \mathsf{Prop}$. We write $\mathsf{Sys} \in \mathcal{A} \iff \mathsf{Adm}(\mathsf{Sys}) = \top$. The predicate is operational only when the following components are independently specified:

$$\mathsf{Adm}(\mathsf{Sys}) \iff C_{\mathsf{acc}}(\mathsf{Sys}) \wedge C_{\mathsf{gen}}(\mathsf{Sys}) \wedge C_{\mathsf{coh}}(\mathsf{Sys}) \wedge C_{\mathsf{bound}}(\mathsf{Sys}),$$

where $C_{\mathsf{acc}}(\mathsf{Sys})$ is structural acceptance; $C_{\mathsf{gen}}(\mathsf{Sys})$ is generative closure; $C_{\mathsf{coh}}(\mathsf{Sys})$ is structural coherence; and $C_{\mathsf{bound}}(\mathsf{Sys})$ is compatibility with the specified boundary/reference structure. No claim is made that admissibility determines semantic truth.

B.3 Structural Acceptance Closure

Let $A(\mathsf{Sys})$ denote the class of admissible configurations of Sys.

Definition B.1 (Acceptance closure)**.** *A system* Sys *satisfies acceptance closure iff* $C_{\mathsf{acc}}(\mathsf{Sys}) \iff \forall X \in A(\mathsf{Sys}),\ A_{\mathsf{Sys}}(X) \in A(\mathsf{Sys})$, *where* $A_{\mathsf{Sys}} : \mathcal{C}_{\mathsf{Sys}} \to \mathcal{C}_{\mathsf{Sys}}$ *is the structural acceptance map.*

Lemma B.2. *If* $C_{\mathsf{acc}}(\mathsf{Sys})$ *holds, then every finite iteration of* A_{Sys} *preserves admissibility:* $X \in A(\mathsf{Sys}) \Rightarrow A^n_{\mathsf{Sys}}(X) \in A(\mathsf{Sys})$ *for all* $n \geq 0$.

Proof. For $n = 0$, the claim is immediate. If $A^n_{\mathsf{Sys}}(X) \in A(\mathsf{Sys})$, then by $C_{\mathsf{acc}}(\mathsf{Sys})$ we have $A^{n+1}_{\mathsf{Sys}}(X) \in A(\mathsf{Sys})$. The result follows by induction. □

B.4 Generative Admissible Expansion

Let $G_{\mathsf{Sys}}(X)$ denote the set of one-step generative extensions of X.

Definition B.3 (Generative admissible expansion)**.** *A system* Sys *satisfies generative admissible expansion iff* $C_{\mathsf{gen}}(\mathsf{Sys}) \iff \forall X \in A(\mathsf{Sys}),\ G_{\mathsf{Sys}}(X) \subseteq A(\mathsf{Sys})$. *Equivalently,* $G_{\mathsf{Sys}}(A(\mathsf{Sys})) \subseteq A(\mathsf{Sys})$.

Lemma B.4. *If* $X \in A(\mathsf{Sys})$ *and* $C_{\mathsf{gen}}(\mathsf{Sys})$ *holds, then every configuration obtained from* X *by a finite sequence of admissible generative transformations remains admissible.*

Proof. This follows directly by induction on the number of generative steps, using $C_{\mathsf{gen}}(\mathsf{Sys})$. □

B.5 Compatibility Coherence

Let K_{coh} denote a binary structural compatibility relation on the admissible configurations, $K_{\mathsf{coh}} \subseteq A(\mathsf{Sys}) \times A(\mathsf{Sys})$; we write $K_{\mathsf{coh}}(X, Y)$ to mean that Y is coherent with X under the coherence/compatibility structure of Sys. The relation is defined on all pairs of admissible configurations (and not only on pairs (X, Y) with $Y \in G_{\mathsf{Sys}}(X)$), so that the transitivity condition below is well-posed.

Definition B.5 (Compatibility coherence)**.** *A system* Sys *satisfies compatibility coherence, written* $C_{\mathsf{coh}}(\mathsf{Sys})$*, iff both of the following hold:*

(i) generated extensions are coherent: $\forall X \in A(\mathsf{Sys}),\ \forall Y \in G_{\mathsf{Sys}}(X) :\ K_{\mathsf{coh}}(X, Y)$;

(ii) transitivity: *for all* $X, Y, Z \in A(\mathsf{Sys})$, $K_{\mathsf{coh}}(X, Y) \wedge K_{\mathsf{coh}}(Y, Z) \Rightarrow K_{\mathsf{coh}}(X, Z)$.

B.6 Coherence of a System

Definition B.6 (Structurally coherent system)**.** *A system* Sys *is called structurally coherent iff* $\mathsf{Coh}(\mathsf{Sys}) \iff C_{\mathsf{acc}}(\mathsf{Sys}) \wedge C_{\mathsf{gen}}(\mathsf{Sys}) \wedge C_{\mathsf{coh}}(\mathsf{Sys})$.

This is a structural coherence condition. It does not assert that the internal statements of Sys are true or that all consequences are semantically valid.

B.7 Coherence Preservation Theorem

Theorem B.7 (Coherence preservation)**.** *Let* Sys *and* Sys′ *be two systems, and let* $T : \mathsf{Sys} \to \mathsf{Sys}'$ *be an admissible transformation satisfying:*

1. $T(A(\mathsf{Sys})) = A(\mathsf{Sys}')$;
2. $T \circ A_{\mathsf{Sys}} = A_{\mathsf{Sys}'} \circ T$;
3. $T(G_{\mathsf{Sys}}(X)) = G_{\mathsf{Sys}'}(T(X))$ *for every* $X \in A(\mathsf{Sys})$;
4. *if* $K_{\mathsf{coh}}(X, Y)$ *holds, then* $K'_{\mathsf{coh}}(T(X), T(Y))$ *holds;*
5. *(*reflection*) if* $K'_{\mathsf{coh}}(T(X), T(Y))$ *holds for* $X, Y \in A(\mathsf{Sys})$*, then* $K_{\mathsf{coh}}(X, Y)$ *holds.*

If $\mathsf{Coh}(\mathsf{Sys})$ *holds, then* $\mathsf{Coh}(\mathsf{Sys}')$ *holds.*

Proof. Because Coh(Sys) holds, we have $C_{\text{acc}}(\text{Sys})$, $C_{\text{gen}}(\text{Sys})$, and $C_{\text{coh}}(\text{Sys})$.

Acceptance closure. Take $Y' \in A(\text{Sys}')$. By condition 1, there exists $X \in A(\text{Sys})$ such that $T(X) = Y'$. From $C_{\text{acc}}(\text{Sys})$, $A_{\text{Sys}}(X) \in A(\text{Sys})$. By condition 2, $T(A_{\text{Sys}}(X)) = A_{\text{Sys}'}(T(X)) = A_{\text{Sys}'}(Y')$. Since $T(A_{\text{Sys}}(X)) \in T(A(\text{Sys})) = A(\text{Sys}')$, it follows that $A_{\text{Sys}'}(Y') \in A(\text{Sys}')$. Hence $C_{\text{acc}}(\text{Sys}')$ holds.

Generative closure. Let $X' \in A(\text{Sys}')$ and $Y' \in G_{\text{Sys}'}(X')$. By condition 1, there exists $X \in A(\text{Sys})$ such that $T(X) = X'$. By condition 3, $G_{\text{Sys}'}(T(X)) = T(G_{\text{Sys}}(X))$. Therefore there exists $Y \in G_{\text{Sys}}(X)$ such that $T(Y) = Y'$. Since $C_{\text{gen}}(\text{Sys})$ holds, $G_{\text{Sys}}(X) \subseteq A(\text{Sys})$, so $Y \in A(\text{Sys})$. Hence $Y' = T(Y) \in T(A(\text{Sys})) = A(\text{Sys}')$. Thus $G_{\text{Sys}'}(X') \subseteq A(\text{Sys}')$, proving $C_{\text{gen}}(\text{Sys}')$.

Coherence, part (i). Let $X' \in A(\text{Sys}')$ and $Y' \in G_{\text{Sys}'}(X')$. As above, there exist $X \in A(\text{Sys})$ and $Y \in G_{\text{Sys}}(X)$ such that $T(X) = X'$ and $T(Y) = Y'$. Since $C_{\text{coh}}(\text{Sys})$ holds, $K_{\text{coh}}(X, Y)$ holds. By condition 4, $K'_{\text{coh}}(T(X), T(Y))$ holds, i.e. $K'_{\text{coh}}(X', Y')$.

Coherence, part (ii) (transitivity of K'_{coh}). Let $X', Y', Z' \in A(\text{Sys}')$ with $K'_{\text{coh}}(X', Y')$ and $K'_{\text{coh}}(Y', Z')$. By condition 1, choose $X, Y, Z \in A(\text{Sys})$ with $T(X) = X'$, $T(Y) = Y'$, $T(Z) = Z'$; the same Y is used for both relations. By condition 5, $K'_{\text{coh}}(T(X), T(Y))$ gives $K_{\text{coh}}(X, Y)$, and $K'_{\text{coh}}(T(Y), T(Z))$ gives $K_{\text{coh}}(Y, Z)$. Since K_{coh} is transitive on $A(\text{Sys})$ by $C_{\text{coh}}(\text{Sys})$(ii), $K_{\text{coh}}(X, Z)$ holds. By condition 4, $K'_{\text{coh}}(T(X), T(Z))$ holds, i.e. $K'_{\text{coh}}(X', Z')$. Hence K'_{coh} is transitive on $A(\text{Sys}')$.

Parts (i) and (ii) give $C_{\text{coh}}(\text{Sys}')$. Combining the three results, $C_{\text{acc}}(\text{Sys}') \wedge C_{\text{gen}}(\text{Sys}') \wedge C_{\text{coh}}(\text{Sys}')$, which is exactly Coh(Sys′). □

Remark B.1 (why condition 5 is needed). Conditions 1–4 alone do not imply transitivity of K'_{coh}, because condition 4 only transports pairs forward from Sys to Sys′ and places no constraint on pairs that K'_{coh} contains beyond those images. A minimal example: let $A(\text{Sys}) = A(\text{Sys}') = \{a, b, c\}$, $G(a) = \{b\}$, $G(b) = \{c\}$, $G(c) = \emptyset$, let $A_{\text{Sys}} = A_{\text{Sys}'} = \text{id}$ and $T = \text{id}$, let $K_{\text{coh}} = \{(a, b), (b, c), (a, c)\}$ (transitive), and let $K'_{\text{coh}} = K_{\text{coh}} \cup \{(c, a)\}$. Conditions 1–4 hold and Coh(Sys) holds, but $K'_{\text{coh}}(b, c)$ and $K'_{\text{coh}}(c, a)$ hold while $K'_{\text{coh}}(b, a)$ does not, so Coh(Sys′) fails. Condition 5 excludes exactly this situation: it requires that K'_{coh} contain no pair of images that is not already coherent in Sys.

B.8 Coherence Is Not Soundness

The theorem above establishes only Coh(Sys) $\Rightarrow$ Coh(Sys′) under the stated transformation conditions. It does not establish Coh(Sys) $\Rightarrow$ Sound(Sys). Soundness requires an independent semantic interpretation $[\![\cdot]\!]_{\text{Sys}}$ and a demonstration that the inference rules and transformations preserve semantic validity. Therefore, in the present framework, Coh(Sys) $\not\Rightarrow$ Sound(Sys).

B.9 Conditions Required for Soundness

Three additional semantic assumptions would be needed for a future soundness result. *H-A — Semantic adequacy:* every admissible structure has a well-defined interpretation, $X \in A(\text{Sys}) \Rightarrow [\![X]\!]_{\text{Sys}}$ is defined. *H-B — Transformation preservation:* for every admissible transformation T, $[\![X]\!]_{\text{Sys}} \Rightarrow [\![T(X)]\!]_{\text{Sys}'}$. *H-C — Inference validity:* if $X \vdash_{\text{Sys}} Y$, then $[\![X]\!]_{\text{Sys}} \models [\![Y]\!]_{\text{Sys}}$. Only if assumptions of this type are added could one formulate a theorem of the form Coh(Sys) $\wedge$ H-A $\wedge$ H-B $\wedge$ H-C $\Rightarrow$ Sound(Sys). No such theorem is claimed.

B.10 CIS and Cross-System Invariance

The CIS constructed in Appendix A, $\text{CIS}(X) = \ln r(X) + \kappa\rho(X)$, is an architectural quantity defined on the structural configuration space. A stronger claim would require a system-level

functional $Q : \mathcal{A} \to \mathbb{R}$ such that for admissibility-equivalent systems, $\text{Sys}_1 \simeq_{\mathcal{A}} \text{Sys}_2 \Rightarrow Q(\text{Sys}_1) = Q(\text{Sys}_2)$. The present axioms do not establish that CIS is such a Q. A meaningful invariant must also not be constant across all admissible systems, i.e. $\exists\, \text{Sys}_1, \text{Sys}_2 : Q(\text{Sys}_1) \neq Q(\text{Sys}_2)$ while $\text{Sys}_1 \simeq_{\mathcal{A}} \text{Sys}_2 \Rightarrow Q(\text{Sys}_1) = Q(\text{Sys}_2)$. The present article establishes the admissibility and coherence structure needed to formulate this test; it does not establish that CIS passes it.

B.11 Status of the Admissibility Layer

Admissibility specifies the admissible domain; CIS is a scalar cost on that domain. Under the conditions of Theorem B.7, structural coherence is preserved by the transformation T; coherence does not imply soundness (B.8).

Appendix C — Architectural Interpretation and Extended Vocabulary

This appendix is non-formal and optional. It provides an interpretive layer for the formal core; the terms introduced here do not enter the axioms, assumptions, or proofs of the main text or Appendix A.

C.1 Terminological Mapping

The present article uses neutral academic vocabulary in its formal core. Earlier documents related to the same research program sometimes use different architectural names for the same or closely related notions. To avoid confusion, the following mapping is provided. The proper name Ahuraic Framework is retained only as the name of the overall architecture (see the Note on Terminology).

Earlier term	Academic equivalent in this article	Brief role
Āshid Framework	Admissibility Framework / meta-level admissibility predicate	admissibility scheme
Asha	Structural Constraint Operator	constraint and acceptance
Spenta	Generative Structural Operator	generation and expansion
Hamakshāni	Structural Coherence Operator	compatibility and coherence
Phon	Primitive Distinction	minimal admissible distinction
Āshāyand	Autonomous Organizational Entity / persistent structural entity	stable structural entity
Filament	Structural Coherence Channel	one-dimensional relational composite
HOSC	Higher-Order Structural Composite	higher-order composite
Sephere	Canonical Architectural Layer	architectural level / coarse-grained structural regime
Ahuraic Space	Organizational State Space / Structural Configuration Space	domain of admissible configurations

C.2 Scope and Status

The following architectural vocabulary is offered only as an organizing interpretation: architectural ground and its three attributes; structural constraint, generative expansion, and coherence; structural coupling as a distinct dependency relation; admissibility scheme; structural configuration space; architectural layers / structural regimes; transition and flux-window conjectures. These notions may be useful for motivating the structural roles of Section 4, but they are not required for the mathematical results concerning CIS, D-CIS, or CAP.

C.3 Architectural Ground and Three Attributes

In the broader architecture, the architectural ground is treated as a precondition for admissible realization, not as an object, field, force, or physical entity. It is the background condition for the possibility of lawful structural organization. The architectural ground is associated with three formal indicators, ordered here as Openness, Unity, and Order:

Attribute	Symbol	Interpretive role
Openness	P_{attr}	generative expansion and novelty
Unity	U_{attr}	coherence and compatibility
Order	O_{attr}	structural order and constraint

The subscripted symbols O_{attr}, P_{attr}, U_{attr} are used only to avoid confusion with the formal symbols O for organization processes, $P_{\text{in}}/P_{\text{out}}$ for D-CIS balance terms, and U otherwise left unspecified in the formal core.

C.4 Structural Constraint, Generative Expansion, and Coherence as Architectural Roles

Interpretive term	Architectural role	Formal counterpart
Structural constraint	constraint and acceptance	admissibility function A_{SO}; refinement relation $\leq$; ρ
Generative expansion	generation and expansion	generative relation G; expansion ratio r
Structural coherence	coherence and compatibility	coherence operator H; coherence conditions

Structural Coupling, in contrast, is a distinct dependency relation represented by Γ_{SC}. It may contribute to coherence or to interaction corrections, but it is not one of the three architectural roles. This mapping does not mean that structural constraint directly "produces" ρ, that generative expansion directly "produces" r, or that coherence alone guarantees additivity of CIS.

C.5 Admissibility Scheme as Meta-Level Admissibility

In the interpretive layer, the admissibility scheme is the regulatory framework that emerges from the interaction of structural constraint, generative expansion, and coherence. It specifies which formal/structural systems are admissible. Formally, this is expressed only at the level of Appendix B: $\text{Adm}(\text{Sys})$. No further ontological status is assigned to the admissibility scheme.

C.6 Structural Configuration Space

The interpretive literature describes an "organizational state space." In the present article, this corresponds to the Structural Configuration Space $(\mathcal{C}, \leq)$ of Section 3. The formal objects are $X \in \mathcal{C}$, $\rho(X)$, $r(X)$, and $\text{CIS}(X)$. Any additional decomposition of the configuration space into components such as $\mathcal{A} = P + D + H + C + I + M$ is interpretive and is not used in the formal theorem. Here $+$ is to be understood as schematic architectural composition, not as a formal direct sum over objects of identical type.

C.7 Architectural Layers and Structural Regimes

The broader architecture uses layered architectural descriptions. In the present article, these may be regarded as domain-specific coarse-grained descriptions of different structural regimes.

They do not define ρ, r, or CIS.

Interpretive layer	Possible structural interpretation
Minimal-distinction layer	minimal distinctions, $\rho \approx 0$
Stable-unit layer	stable elementary units, $\rho > 0$
Relational-composite layer	compressed composite structures
Higher-order-composite layer	higher-order composites
High-constraint nodal layer	high-constraint nodal structures
Emergent-continuous regime	emergent continuous regime
Stable-pattern regime	stable local or distributed configurations

C.8 Transition, Flux Window, and Dynamical Conjectures

The exploratory NSQ record (Section 9) suggests that many systems exhibit flow-limited persistence, threshold-like transition, and compensatory balance. A candidate dynamical normal form motivated by these motifs, with a two-sided flux window $J_{\text{min}} < J < J_{\text{max}}$, is given in Appendix E; it is an interpretive and empirical conjecture, not a formal result of the present article, and is not derived from Theorem A.1.

C.9 Relation to the Main Text

The formal core of the article remains: Structural Configuration Space $\rightarrow$ (r, ρ) $\rightarrow$ CIS $\rightarrow$ D-CIS $\rightarrow$ CAP. The architectural interpretation provides a possible vocabulary for reading this chain, but it does not modify the mathematical status of the constructs or the canonical form $\text{CIS}(X) = \ln r(X) + \kappa\rho(X)$. In particular, the article does not claim that the emergence of space-time or physical fields is a direct consequence of $\delta\text{CIS} = 0$, nor that the architectural ground, admissibility scheme, structural constraint, generative expansion, coherence, or coupling are physical entities.

Appendix D — Qualitative NSQ Mapping

The table below maps recurring qualitative descriptors identified in the 120-case NSQ comparative record (Jalali et al., 2026a) to the architectural constructs introduced in this article. The mapping is conceptual and interpretive; it does not treat the NSQ record as a numerical dataset and does not imply dimensional equivalence between cases.

Recurring NSQ descriptor	Architectural interpretation	Associated construct(s)	Status
Flow or throughput	Domain-specific enabling flux, resource supply, information rate, matter flow, or energy throughput	J, J_{crit}, C	Conceptual candidate for operationalization
Constrained configuration	Admissible structural arrangements, restrictions on possible states, and structural boundary conditions	Structural Order (SO), $\mathcal{C}$, CIS	Descriptive correspondence

Recurring NSQ descriptor	Architectural interpretation	Associated construct(s)	Status
Dynamic compensation	Balance between processes increasing structural cost and processes reducing, exporting, or dissipating it	D-CIS, P_{in}, P_{out}	Domain-specific modeling
Threshold-like transition	Crossing of a critical support, control, coupling, or resource condition associated with persistence or regime change	J_{crit}, C, CAP	Hypothesis-generating correspondence
Hierarchical refinement	Increased depth of admissible construction, differentiation, nesting, or structural layering	Organizational Rank $\rho(X)$, CIS	Candidate proxy relationship
Coupling among components or levels	Dependency, interaction, regulation, or constraint relations among elements and structural levels	Structural Coupling (SC), Γ_{SC}, J_{crit}	Descriptive correspondence
Formation, growth, or diversification	Expansion of accessible configurations under a specified generative rule	Structural Generativity (SG), $G(X)$, $r(X)$	Candidate operational relationship
Persistence despite disturbance	Continued occupancy of an admissible structural regime under bounded perturbation	CAP, attractor-landscape descriptors	Requires independent empirical test
Collapse, dissolution, or irreversible transition	Loss of an admissible configuration, structural reorganization, or transition to another regime	ΔCIS, CAP, domain-specific dynamics	Descriptive correspondence only

Note on interpretation. The NSQ descriptors do not define the formal quantities used in the article. For example, a qualitative observation of "flow" does not by itself define the enabling flux J; a hierarchical description does not automatically measure Organizational Rank ρ; and a threshold-like transition does not establish the CAP condition $C = J/J_{\text{crit}}(\rho)$. Any empirical application must independently specify: (1) the system boundary and descriptive resolution; (2) the admissible configuration space $\mathcal{C}$; (3) the relevant structural measure and generative relation; (4) the operational proxy for Organizational Rank; (5) the enabling flux J; (6) the procedure for estimating $J_{\text{crit}}(\rho)$; and (7) the persistence or transition outcome to be predicted.

Appendix E — NSQ-Derived Inductive Motif (Not Part of the Formal Core)

This appendix records a recurring structural motif that emerged from the exploratory NSQ comparative record and from subsequent qualitative synthesis. It is a hypothesis-generating pattern; it is not part of the formal architecture of Sections 3–7 and is not derived from Theorem A.1.

E.1 Two Structural Coordinates

Let J denote a normalized enabling flux, and let Ψ denote a coarse-grained organizational intensity or active structural order parameter. These symbols are introduced only for the present appendix and are not assumed to coincide with J_{crit}, ρ, or C used in the main text.

E.2 Effective Potential

Across many NSQ cases, persistence appears to depend on the position of J relative not only to a lower critical flux but also to an upper critical flux. This suggests an effective potential of the form

$$V(\Psi; J) = \tfrac{1}{2}\, g(J)\, \Psi^2 + \tfrac{b}{4}\, \Psi^4, \qquad b > 0, \tag{E.1}$$

with

$$g(J) = \gamma J^2 - a(J - J_c), \qquad a, \gamma > 0. \tag{E.2}$$

Here J_c is a lower critical flux scale.

E.3 Flux Window and Attractor

A nontrivial structural attractor exists when

$$g(J) < 0. \tag{E.3}$$

The quadratic $g(J)$ is convex because $\gamma > 0$. Therefore it takes negative values between its two real roots. The condition for the existence of a finite flux window is

$$a > 4\gamma J_c. \tag{E.4}$$

Under this condition, the roots are

$$J_{\text{min}} = \frac{a - \sqrt{a^2 - 4\gamma a J_c}}{2\gamma}, \qquad J_{\text{max}} = \frac{a + \sqrt{a^2 - 4\gamma a J_c}}{2\gamma}, \tag{E.5}$$

and the finite window is $J_{\text{min}} < J < J_{\text{max}}$. Within this window, the nontrivial stable state for a nonnegative order parameter is

$$\Psi^* = \sqrt{\frac{-g(J)}{b}}. \tag{E.6}$$

Outside the window, $g(J) > 0$, the nontrivial attractor disappears, and the only stable state is the collapsed state $\Psi = 0$.

E.4 Stochastic Attractor Dynamics

As a minimal dynamical expression of this motif, one may consider the Langevin-type equation

$$\frac{d\Psi}{dt} = -g(J)\Psi - b\Psi^3 + \sigma\, \xi(t), \tag{E.7}$$

where $\sigma \geq 0$ is a noise amplitude and $\xi(t)$ is a standard white-noise process. Equation (E.7) is a compact summary of the observed recurrence of threshold-limited persistence, destabilization under excessive or insufficient flux, and noise-sensitive transitions; it is not proposed as a law of motion.

E.5 Status and Boundary

The parameters a, b, γ, and σ are not estimated here, and the window form is an inductive motif from the NSQ record, not a prediction of the architecture in the main text. Any future attempt to promote this motif to a formal model would require: (1) independent operationalization of Ψ and J; (2) an explicit derivation or calibration of the effective potential $V(\Psi; J)$; (3) separation of training and test data; (4) comparison with appropriate baseline models; and (5) propagation of uncertainty in all estimated quantities. Until those conditions are met, this appendix remains a candidate extension and not a component of the Ahuraic Framework as defined in the main text.

Table 1: Conceptual comparison between CIS and established measures.

Measure/Concept	**What it measures**	**Relation to CIS**
Shannon entropy	Uncertainty over probability distributions	Not equivalent; CIS is a structural cost functional.
Landauer cost	Physical dissipation associated with logical erasure	Conceptually related; CIS is not a fixed physical energy.
Thermodynamic entropy	Statistical multiplicity of physical microstates	Not equivalent; may correlate only under an explicit mapping.
Free energy	Thermodynamic energetic potential of a system	Distinct; correspondence would require an explicit mapping or affine energy relation.
Kolmogorov complexity	Shortest program length	Conceptually related; CIS is not algorithmic complexity.
Statistical complexity	Stored predictive information in computational mechanics	Complementary; CIS adds a structural rank term.
Logical depth	Computational time associated with generating an object	Conceptually related; CIS is not computational depth.
Integrated information (Φ)	Irreducible causal integration in causal systems	Distinct; CIS is a structural cost.
Network modularity	Graph-based organization	Domain-specific proxy only.
Correlation length / entanglement entropy	Spatial or quantum correlations	Possible operational proxies for ρ in suitable domains, but not identical to ρ.
Self-organization	Emergence of collective order in nonequilibrium systems	Complementary; CIS is architectural.
Autopoiesis / systems theory	Self-production and coupling	SC generalizes the coupling relation; SH is a coherence role.
Constraint closure	Mutual dependence of constraints	SO/SG/SH broaden the constraint description.
Free-energy principle	Variational free energy and adaptation	CAP is different; it does not require Bayesian inference.
Attractor capacity	Mathematical capacity associated with dynamical attractors	CAP uses the word capacity operationally and is not identical to this mathematical notion.